Solution for Problem B5

(10 points) The probability of occupation of the levels in the metal are given by the Fermi Dirac distribution

$$p = \frac{1}{\exp[\beta(E-\mu)]+1}.$$

This holds for the electrons outside the metal as well, where

$$E - \mu = \Phi + \frac{\hbar^2 k^2}{2m},$$

and $\mathbf{k}$ is the wavevector of a free electron. The density of such states is $V d^3k/(2\pi)^3$, where V is the volume occupied by the free electrons. The total number of electrons outside the metal is

$$N = 2\frac{V}{(2\pi)^3}\int p\, d^3k.$$

The factor 2 comes from the two spin orientations allowed for each spatial state. Since $\beta\Phi >> 1$, the Fermi-Dirac distribution becomes Maxwellian, and

$$N = 2\frac{V}{(2\pi)^3}\exp(-\beta\Phi)4\pi\int_0^\infty k^2 dk \exp(-\beta\hbar^2k^2/2m),$$

$$n = \frac{N}{V} = \frac{2}{h^3}(2\pi mkT)^{3/2}\exp[-\Phi/kT].$$