

where $\epsilon^{(1)}$ is an energy eigenvalue, and the a_i 's characterize the corresponding eigenstate. The *characteristic equation* for the eigenvalues is

$$-[\epsilon^{(1)}]^3 + 2M^2\epsilon^{(1)} = 0.$$

The three roots are $\epsilon^{(1)} = -M\sqrt{2}, 0, +M\sqrt{2}$. The corresponding eigenvectors and their eigenvalues through first order are:

$$\begin{array}{ll} |A\rangle - \sqrt{2}|B\rangle + |C\rangle & \epsilon - M\sqrt{2}, \\ |A\rangle - |C\rangle & \epsilon, \\ |A\rangle + \sqrt{2}|B\rangle + |C\rangle & \epsilon + M\sqrt{2}. \end{array}$$