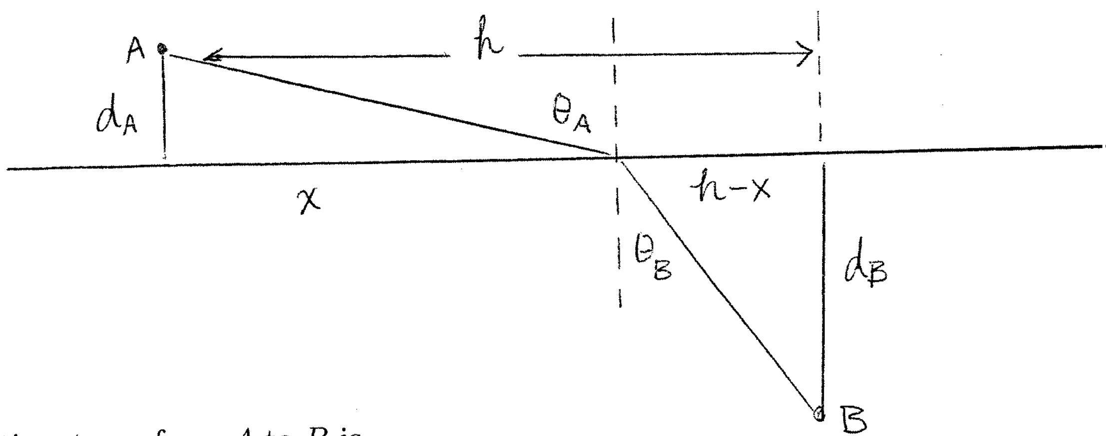
Solution for Problem B2

a) (5 points) Possible paths between A and B are parametrized by the coordinate x at which the rays meet the interface.



The time to go from A to B is

$$T = \frac{n_A}{c}\sqrt{d_A^2 + x^2} + \frac{n_B}{c}\sqrt{d_B^2 + (h-x)^2}.$$

This is minimized when

$$0 = \frac{dT}{dx} = \frac{n_A}{c}\frac{x}{\sqrt{d_A^2 + x^2}} - \frac{n_B}{c}\frac{h-x}{\sqrt{d_B^2 + (h-x)^2}}.$$

note that

$$\sin\theta_A = \frac{x}{\sqrt{d_A^2 + x^2}}; \qquad \sin\theta_B = \frac{h-x}{\sqrt{d_B^2 + (h-x)^2}}.$$

Thus

$$n_A \sin\theta_A = n_B \sin\theta_B.$$

This is Snell's law, which governs refraction of the physical rays at the interface. The minimum time path is the physical path followed by the rays.

b) (2 points) The expression for T in the inhomogeneous case depends on the parametrization of the path. One possibility is to express the y and z coordinates of the path as functions of x:

$$y(x): \quad \begin{cases} y(x_A) = y_A, \\ y(x_B) = y_B. \end{cases} \qquad z(x): \quad \begin{cases} z(x_A) = z_A, \\ z(x_B) = z_B. \end{cases}$$