Solution for Problem B1

a) (2 points) The potential corresponding to constant force F is $V(x) = -Fx$. With this potential, motion is unbounded as $x \to \infty$. As a result, there is no quantization of E, and the spectrum is entirely continuous. (This assumes $F > 0$. When $F < 0$, motion is unbounded at negative x, but the conclusion is the same.)

b) (2 points) The potential $V(x)$ has no minimum, so there is no minimum for the spectrum of E. There is no maximum for E for motion governed by a potential, so we find energy eigenvalues in the range $-\infty$ to $+\infty$.

c) (2 points) The time-independent Schrödinger equation is a second order ordinary differential equation. For each energy E it has two solutions. However, only one of these remains bounded as $x \to -\infty$, in the classically forbidden region. Thus there is one admissible solution for each E, and the states are non-degenerate.

d) (2 points) The time-dependence of the expectation of operator A is

$$\frac{d\langle A\rangle_t}{dt} = \frac{1}{i\hbar}\langle[A, H]\rangle_t.$$

(See Liboff, p. 174.) We find $[p, H] = -F[p, x] = i\hbar F$. Thus,

$$\frac{d\langle p\rangle_t}{dt} = F; \qquad \langle p\rangle_t = \langle p\rangle_0 + Ft.$$

Classically, Newton's second law has the solution $p(t) = p(0) + Ft$, so the classical and quantum time evolutions are the same.

e) (2 points) We find

$$[x, H] = \frac{1}{2m}[x, p^2] = \frac{1}{2m}\left([x, p]p + p[x, p]\right) = \frac{i\hbar p}{m}, \qquad \frac{d\langle x\rangle_t}{dt} = \frac{\langle p\rangle_t}{m} = \frac{\langle p\rangle_0}{m} + \frac{Ft}{m}.$$

The solution of this equation is

$$\langle x\rangle_t = \langle x\rangle_0 + \frac{\langle p\rangle_0 t}{m} + \frac{Ft^2}{2m}.$$

Since classically we have constant acceleration motion, $x(t) = x(0) + p(0)t/m + Ft^2/2m$, and the classical and quantum evolutions are the same.