

Altogether,

$$\frac{d}{dt}(\omega^p E) = \frac{\omega^p \dot{\ell}}{\ell} \left[\left(-\frac{p}{2} - 2 \right) T + \left(-\frac{p}{2} + 1 \right) V + m\ell \ddot{\ell} \right].$$

The expression within the bracket must vanish after integrating over a cycle of oscillation. In doing this we can drop terms proportional to $\dot{\ell}$ and $\ddot{\ell}$, which are small compared to the other terms in T and V . For a harmonic oscillator, the average kinetic and potential energies are equal over a cycle of oscillation. Hence the condition on p is

$$-\frac{p}{2} - 2 = -\left(-\frac{p}{2} + 1 \right),$$

or $p = -1$.