Solution for Problem A2

a) (5 points) The following 3 equations express conversation of momentum, angular momentum and energy

$$mv_o = mv_f + M\frac{v_u + v_l}{2}$$
$$m(v_o - v_f)\ell = \frac{1}{12}M4\ell^2\frac{v_u - v_l}{2\ell}$$
$$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}(\frac{1}{3}M\ell^2)(\frac{v_u - v_l}{2\ell})^2 + \frac{1}{2}M(\frac{v_u + v_l}{2})^2$$

Here v_u andv_l denote the velocities of the upper and lower end of the plank. Solving for v_f we get

$$v_f = \frac{4\frac{m}{M} - 1}{4\frac{m}{M} + 1}v_o$$

b) (5 points) Now only angular momentum (about the pivot) and energy are conserved:

$$mv_o 2\ell = mv_f 2\ell + \tfrac{4}{3}M\ell^2\omega$$
$$\tfrac{1}{2}mv_o^2 = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}\left(\tfrac{4}{3}M\ell^2\right)\omega^2$$

solving for v_f we get

$$v_f = \frac{3\frac{m}{M} - 1}{3\frac{m}{M} + 1}v_o$$