C6) Hot stars radiate a large number of photons in the ultraviolet with enough energy to ionize atomic hydrogen from the ground state. Thus an HII region forms, known is a "Strömgren sphere," around a hot star in which all the hydrogen is ionized. (In this problem, you should neglect the expansion of the HII region caused by its excess thermal pressure over that in the surrounding gas.)

a) (3 points) The ionized gas recombines at a rate $\alpha n_e n_p$ per unit volume, where n_e and n_p are the numbers of electrons and protons per unit volume, and α is the recombination probability per second for the given densities. Assuming the surrounding gas is pure atomic hydrogen of a uniform density, and that the star emits N Lyman continuum photons per second ($\lambda <$ 912Å), derive a formula for the steady state radius of the Strömgren sphere of ionized hydrogen.

b) (2 points) Make a rough estimate for N, assuming an O6 star with an effective surface temperature of 45 000 K and a bolometric luminosity of 5×10^{38} erg s^{-1}. (Wien's Law, $\lambda_{\rm max} = 0.29/T$ cm with T in Kelvin, may be useful.)

c) (1 point) Evaluate the Strömgren radius, assuming the surrounding gas has a density of 1000 H atoms cm^{-3}. Take $\alpha \simeq 3 \times 10^{-13}$ cm^3 s^{-1}.

d) (2 points) Recombinations occur to any bound state of the hydrogen atom, and may be followed by a cascade of transitions towards the ground state. Given that the HII region is surrounded by ground state atomic hydrogen, what will be the fate of re-emitted photons in (i) the Lyman series or continuum, and (ii) Balmer and higher series lines? Why, therefore, does the value of α given above include all possible recombinations *except* those to the ground state?

e) (2 points) In practice, interstellar gas is mixed with dust which eventually absorbs the Lyman photons and reradiates the energy in the infrared. Give a qualitative description of the spectrum emitted by an HII region, focusing on the optical and infrared.