C5) Alkali metal atoms have one electron outside a closed shell. When these form solids, assume that the kinetic energy of these valence electrons can be calculated in a free electron approximation, and that the (Coulomb) potential energy of the *whole system* is approximately inversely proportional to the nearest neighbor spacing between the nuclei which form a body centered cubic structure. Define what system of units you are using; in choosing, you may wish to consider that you will be asked for final answers in Rydberg energy units, and Bohr length units.

a) (2 points) Find the kinetic energy per unit volume in terms of the Fermi wave vector k_{F}.

b) (2 points) Find the number of valence electrons per unit volume in terms of k_{F}, and thereby find the kinetic energy per electron.

c) (2 points) Using the assumption in the preamble above, write the potential energy as a function of k_{F} and an unknown constant α.

d) (2 points) Suppose we use the experimentally known density of the equilibriumbulk solid (at zero absolute temperature) and determine that $k_{\mathrm{F}} = 0.5$ Bohr^{-1}. Use this information to predict the total energy per electron in Rydberg energy units.

e) (2 points) Suppose it is known experimentally that the ionization energy of a single atom of this material is 0.3 Rydberg. Use this fact to discuss the accuracy of your answer to part d).