C4) Consider a nonmagnetic metal whose frequency dependent dielectric function (a diagonal tensor in this problem) can be approximated, for sufficiently small wave vectors q, by

$$\epsilon(\omega) = 1 - \frac{\omega_{\rm p}^2}{\omega(\omega + i/\tau)} \tag{1}$$

where $\omega_{\rm p}$, the plasma frequency, and τ, a relaxation time, are constants such that $\omega_{\rm p}\tau \gg 1$, and ω is the angular frequency. As used in condensed matter physics, ϵ is a dimensionless quantity defined by $(4\pi k_e)\vec{P} = (\epsilon - 1)\vec{E}$ when the electric field E and polarization P vary $\propto \exp(i\vec{q}\cdot\vec{r} - i\omega t)$. The quantity k_e is the constant in Coulomb's law, typically either unity (Gaussian) or 9×10^{11} Vm/C (SI). Note also, a time dependent polarization and an induced current density are indistinguisable, so that all conductance effects are implicitly included in ϵ, and the inclusion of a separate induced current term in Maxwell's equations would be redundant.

a) (2 points) Find the frequency of longitudinal wave propagation as a function of wave vector q. Neglect any correction of order $1/\omega_{\rm p}\tau$ or smaller.

b) (2 points) Find the frequency of transverse wave propagation as a function of wave vector q. Neglect any correction of order $1/\omega_{\rm p}\tau$ or smaller.

Please turn over