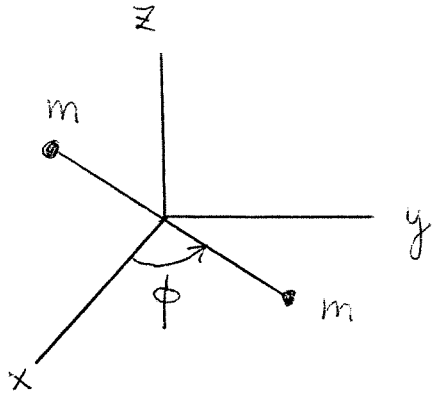
B4) A quantum mechanical plane rotator consists of two rigidly connected particles of mass m rotating in the x-y plane about their center of mass, as shown below. The rod of length d connecting the particles has negligible mass.



a) (3 points) Write down the system's Hamiltonian in terms of m, d, and the angle ϕ.

b) (3 points) Suppose the initial state of the rotator is given by

$$\psi(\phi, t = 0) = A \cos^2 \phi,$$

where A is a constant. Consider a single measurement of the angular momentum L_z in this state.

- What are the possible values of L_z observed in that measurement?
- What are the probabilities for each value?
- What is the expectation value of L_z^2 in this state?
- What is the expectation value of the total energy of the rotator in this state?

c) (2 points) Find $\psi(\phi, t)$ for $t > 0$, given the initial state in Part b). What is the angular frequency, ω_1, of the periodic time dependence of $\psi(\phi, t)$?

d) (1 point) Suppose that at $t = 0$, the distance between the particles of the plane rotator collapses suddenly to $d/2$. What is the new Hamiltonian for $t > 0$? Solve for $\psi(\phi, t)$ for $t > 0$. Answer the questions of part b) above for this new state.

e) (1 point) What is the new angular frequency, ω_2, for $t > 0$?