B3) Consider the three-level system defined by the Hamiltonian, H_0 with *distinct* eigenvalues, ϵ_A, ϵ_B, and ϵ_C, corresponding to the eigenstates $|A\rangle$, $|B\rangle$, and $|C\rangle$, respectively. This system is subject to a time-independent perturbation, H_1 with real matrix elements:

$$\langle A|H_1|A\rangle = \langle B|H_1|B\rangle = \langle C|H_1|C\rangle = 0,$$

and

$$\langle A|H_1|B\rangle = \langle B|H_1|C\rangle = M,$$

and

$$\langle A|H_1|C\rangle = 0.$$

a) (4 points) Calculate the energy eigenvalues of the full Hamiltonian, $H_0 + H_1$, to second order in M (i.e., including terms of order M^2).

b) (3 points) Calculate the corrections to the state vectors $|A\rangle$, $|B\rangle$, and $|C\rangle$, to order M.

c) (3 points) Suppose now that the unperturbed states are degenerate, so that $\epsilon_A = \epsilon_B = \epsilon_C \equiv \epsilon$. Calculate the eigenvalues and eigenvectors of the full Hamiltonian to leading order in M.

You do not need to worry about the normalization of your new eigenstates in parts (b) and (c) above.