A4) Consider a simple pendulum executing small oscillations. The string is shortened an amount $\Delta\ell$ very slowly so that the change in amplitude from one oscillation to the next is negligible.

a) (2 points) Express the kinetic energy of the pendulum in terms of the angle of deflection, θ, the length of the string ℓ and $\dot{\theta}$ and $\dot{\ell}$.

b) (2 points) Express the potential energy in terms of the same variables. Make the usual approximation that applies when $\theta << 1$ radian.

c) (2 points) Find the Lagrangian of the pendulum, and from that its equation of motion. Check that your answer agrees with the expected one when $\dot{\ell} = 0$.

Under these conditions the expression $\omega^p E$ remains constant as the length of the string is changed. Here ω is the angular frequency of oscillation, E is the total energy of the pendulum, and p is a constant to be determined.

d) (4 points) Determine p by computing $d(\omega^p E)/dt$ and using it to show that $\omega^p E$ does not change over one cycle of oscillation, provided p is appropriately chosen.

Thus, if $\dot{\ell}$ is so small that ℓ can be regarded as constant over an oscillation, $\omega^p E$ becomes an *adiabatic invariant* of the pendulum's motion. This adiabatic invariant can be used to relate the amplitude of oscillation after the string length is changed to that before.