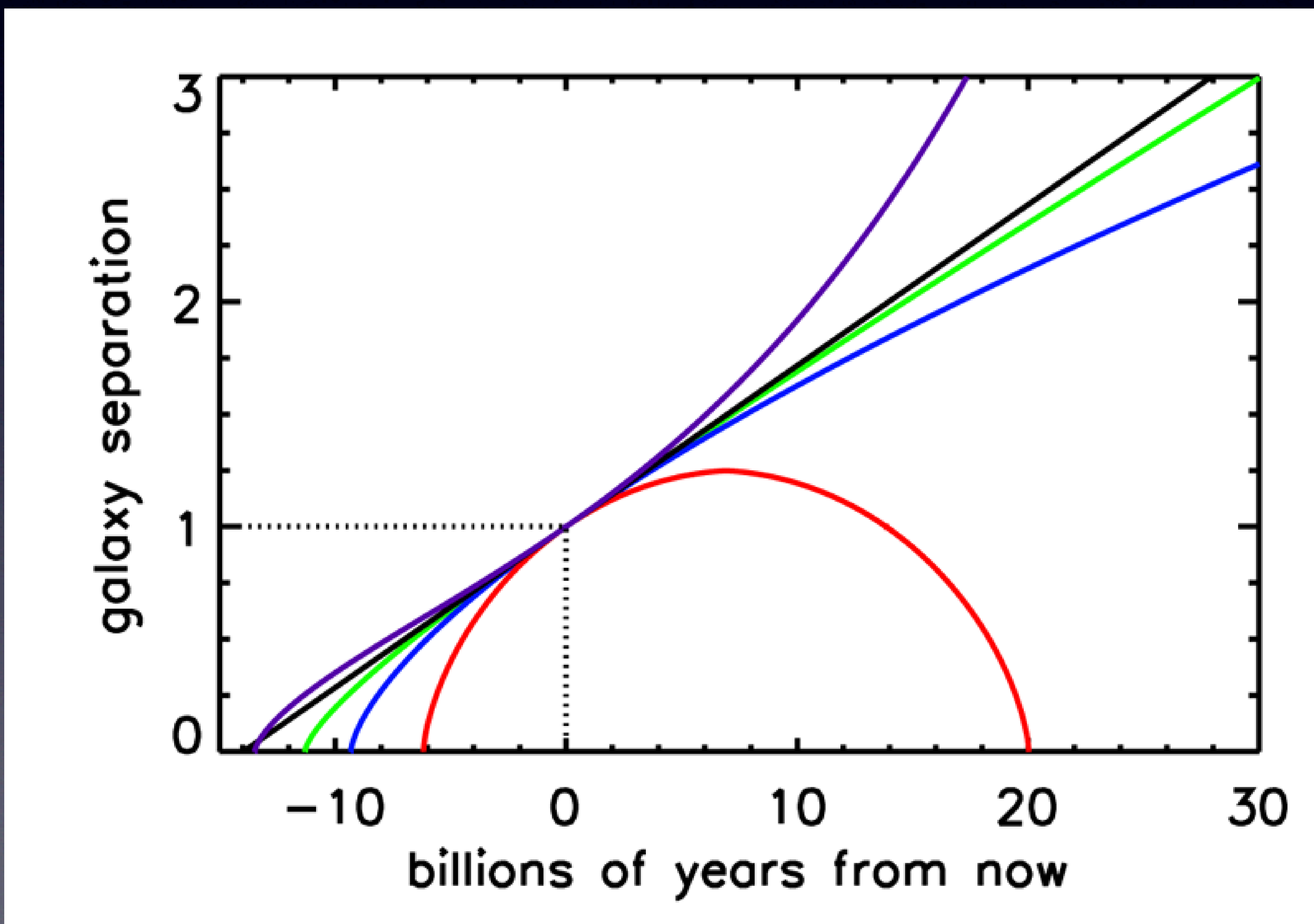
The Cosmological Constant

Carroll, Press, & Turner *ARAA* **30**, 499 (1992)

Physics 690 – Prof. S. Jha – 31 January 2008

Introduction

- Nebulae are other galaxies
- They are distributed isotropically
- Redshift increasing with distance
- Ergo, expanding universe
- Not in accord with popular prejudice for a steady state universe
- Prompts Einstein to add cosmological constant, then abandon it
- Cosmological constant now back in style



$$z = \frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} - 1$$

$$1 + z = \frac{1}{a} = \frac{R_0}{R}$$

Introduction

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_M + \frac{\Lambda}{3} - \frac{k}{a^2}$$

H – Hubble constant
a – Scale factor
k = {-1, 0, +1}
– Curvature parameter

Isotropy + Homogeneity ⇒ GR = FRW universe

Evolution described by above equation.

Take present day values H_0 and $a_0 = 1$, and divide through by H_0.

Define new quantities:

$$\Omega_M \equiv \frac{8\pi G}{3H_0^2}\rho_{M_0} \qquad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \qquad \Omega_k \equiv -\frac{k}{H_0^2}$$

so that $\Omega_M + \Omega_\Lambda + \Omega_k = 1$

- In non-gravitational physics, only energy *differences* are relevant. (Zero-point of potential energy may be selected arbitrarily; kinetic energy may be eliminated by boosting to rest frame.)
- But in GR, curvature couples directly to the energy-momentum tensor! $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- The universe is affected by the ground state of all non-gravitational fields.

- QM tells us that the ground state energy of a quadratic degree of freedom is $E_0 = \hbar\omega/2$

- In QFT we have a continuum of states whose integral diverges!

- Choose a small scale cutoff where we lose confidence in the theory: Planck scale 10^{19} GeV $\Rightarrow$ vacuum energy density ~ 10^{92} erg/cm^3

- 120 orders of magnitude too large!

- “Bare” cosmological constant that cancels ground state energies?
  Given the large number of fields, that would be quite a coincidence.

- Also, inflationary models are driven by some sort of cosmological constant, so whatever mechanism makes it small today must also allow it to be large in the past.

Expansion Dynamics

Rewrite the first equation as

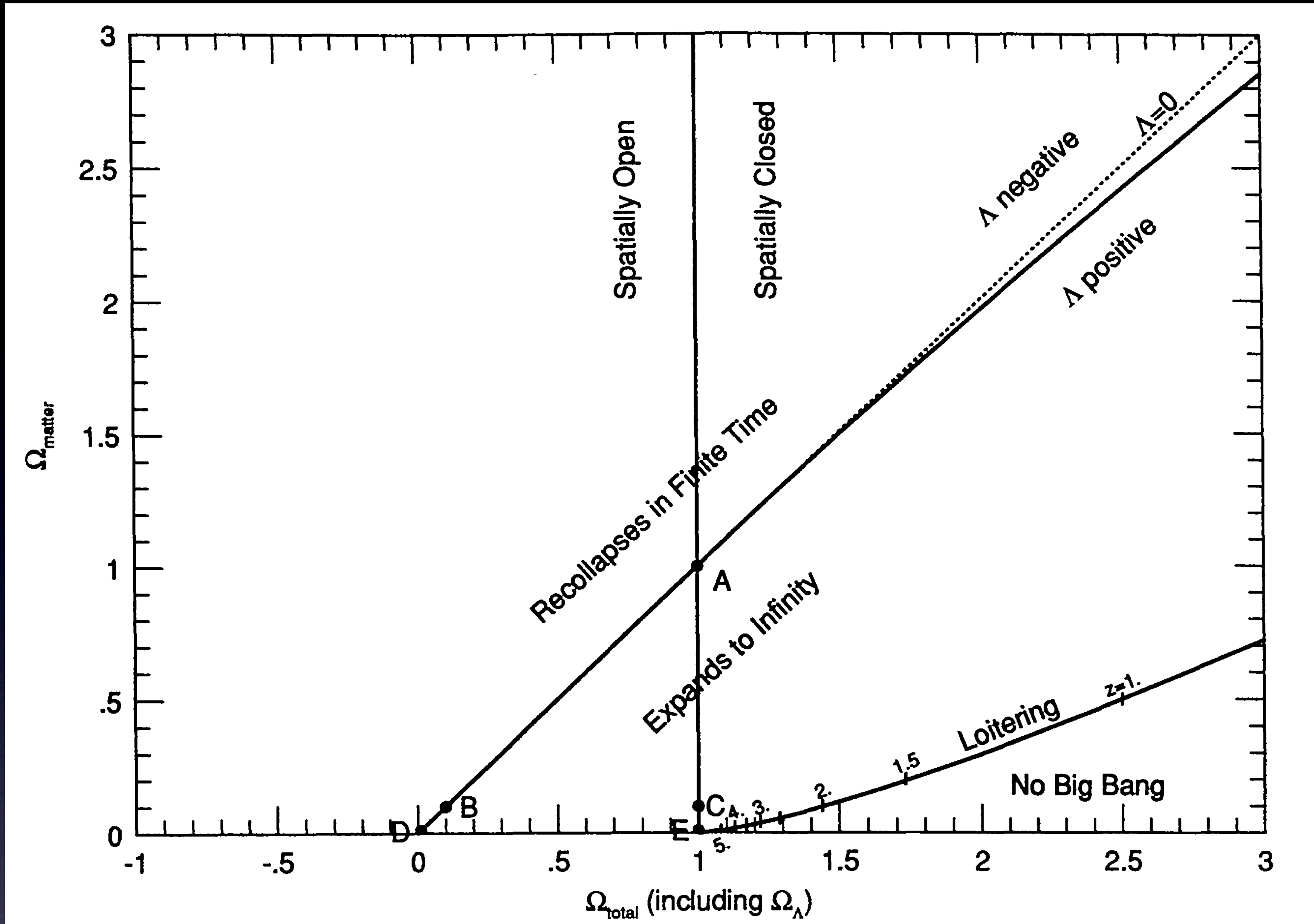
$$\left(\frac{da}{d\tau}\right)^2 = 1 + \Omega_M\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1)$$

Alternatively, we may cast things in terms of Ω_M and

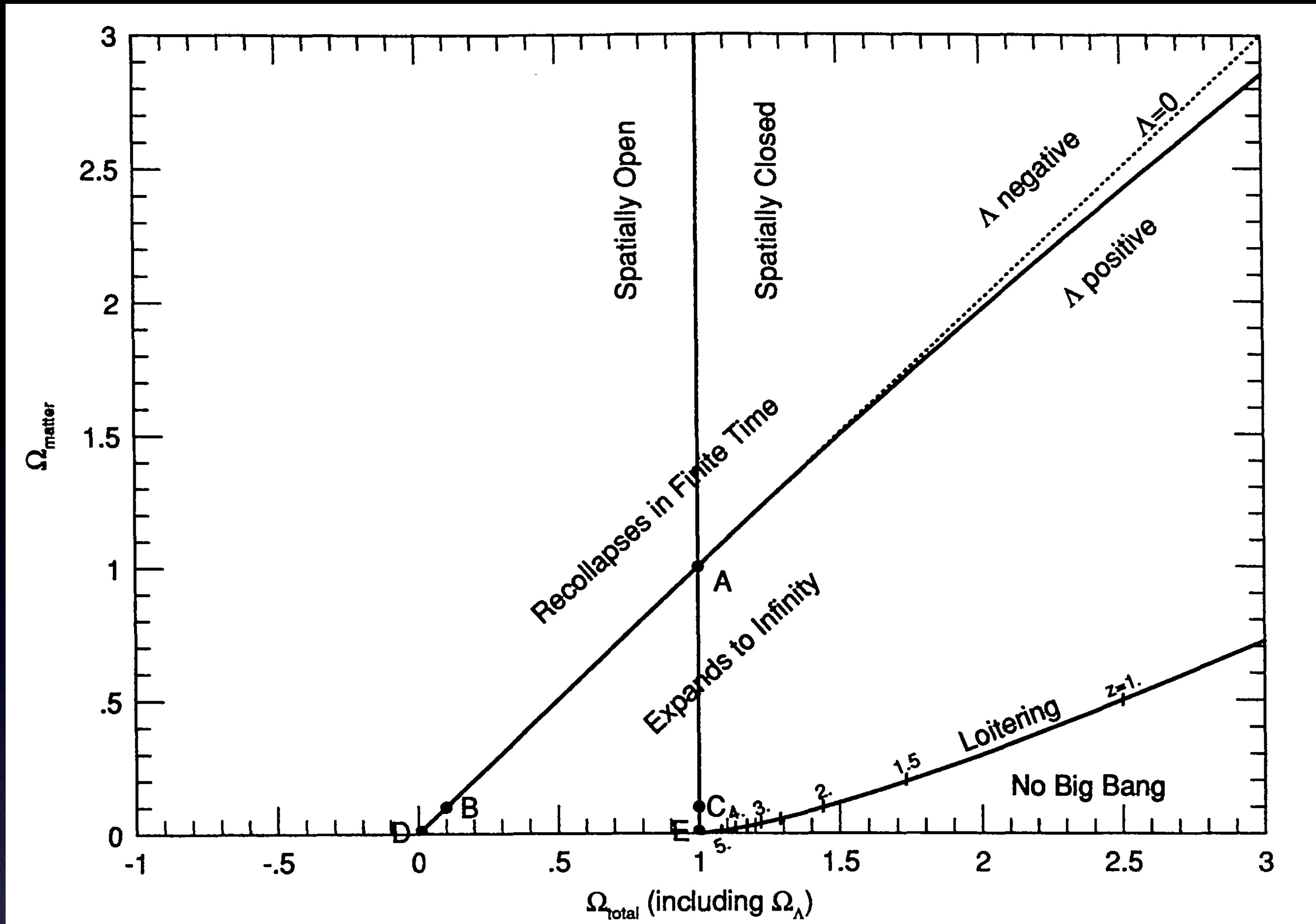
$$\Omega_{\rm tot} \equiv \Omega_M + \Omega_\Lambda = 1 - \Omega_k$$

$\Omega_{\rm tot} > 1$ – closed universe

$\Omega_{\rm tot} < 1$ – open universe

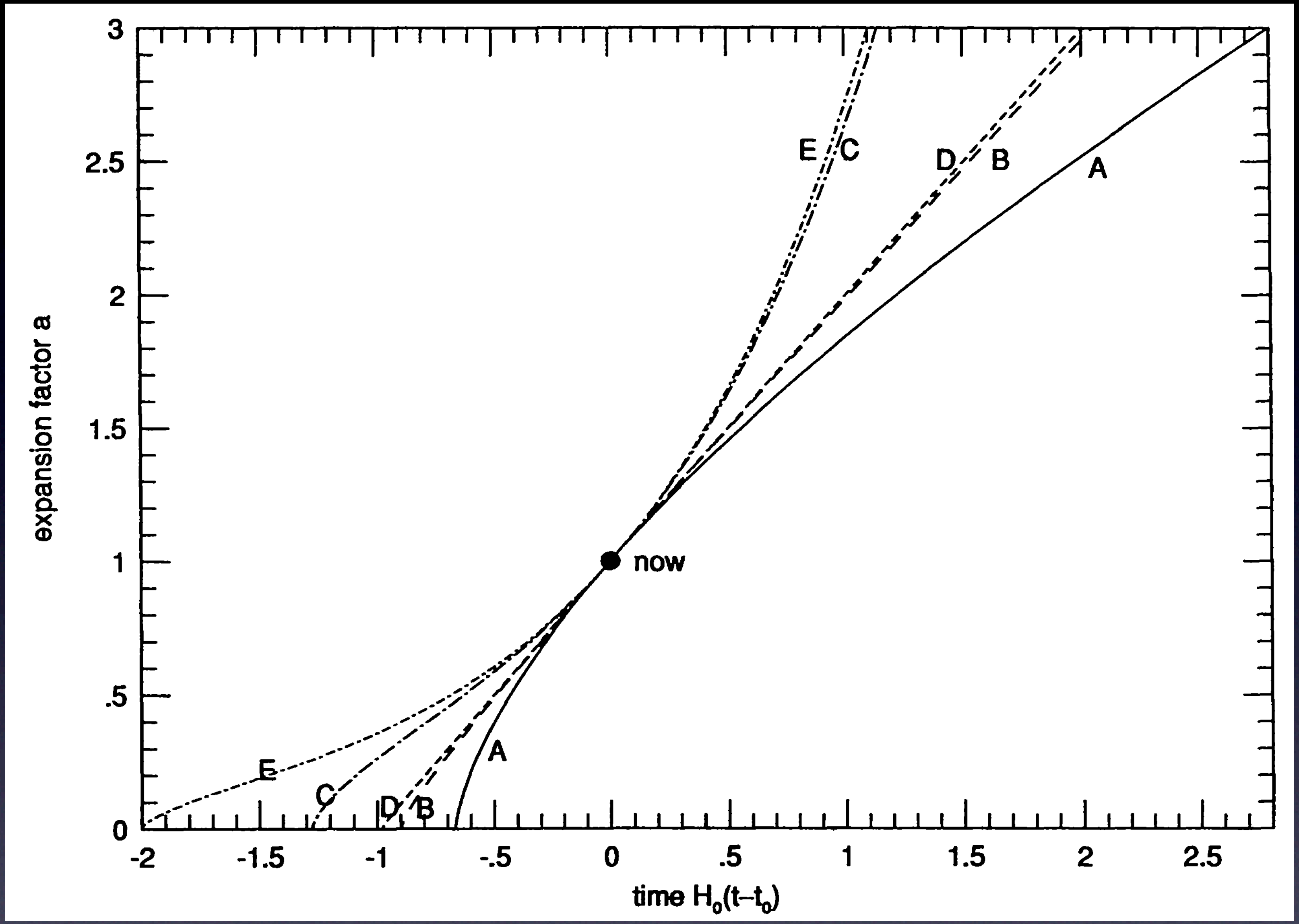


- Recollapse inevitable for $\Lambda < 0$, since it works in the same direction as gravity.
- If $\Lambda > 0$ and if Ω_M is *not too large*, the universe will expand forever (asymptotically deSitter).



- For *very large* Ω_Λ , $a(\tau) \neq 0$ for any times! Easy to test: we should see no objects with redshift greater than $z_{\max} = 1/a_{\min} - 1.$ Ruled out by high redshift quasars and the CMB.
- "Loitering universe:" limiting case of bouncing universe.

Expansion History



Model	Ω_{tot}	Ω_M	Ω_Λ	Description
A	1	1	0	flat, matter dominated, no Λ
B	0.1	0.1	0	open, plausible matter, no Λ
C	1	0.1	0.9	flat, Λ plus plausible matter
D	0.01	0.01	0	open, minimal matter, no Λ
E	1	0.01	0.99	flat, Λ plus minimal matter

Lookback Time

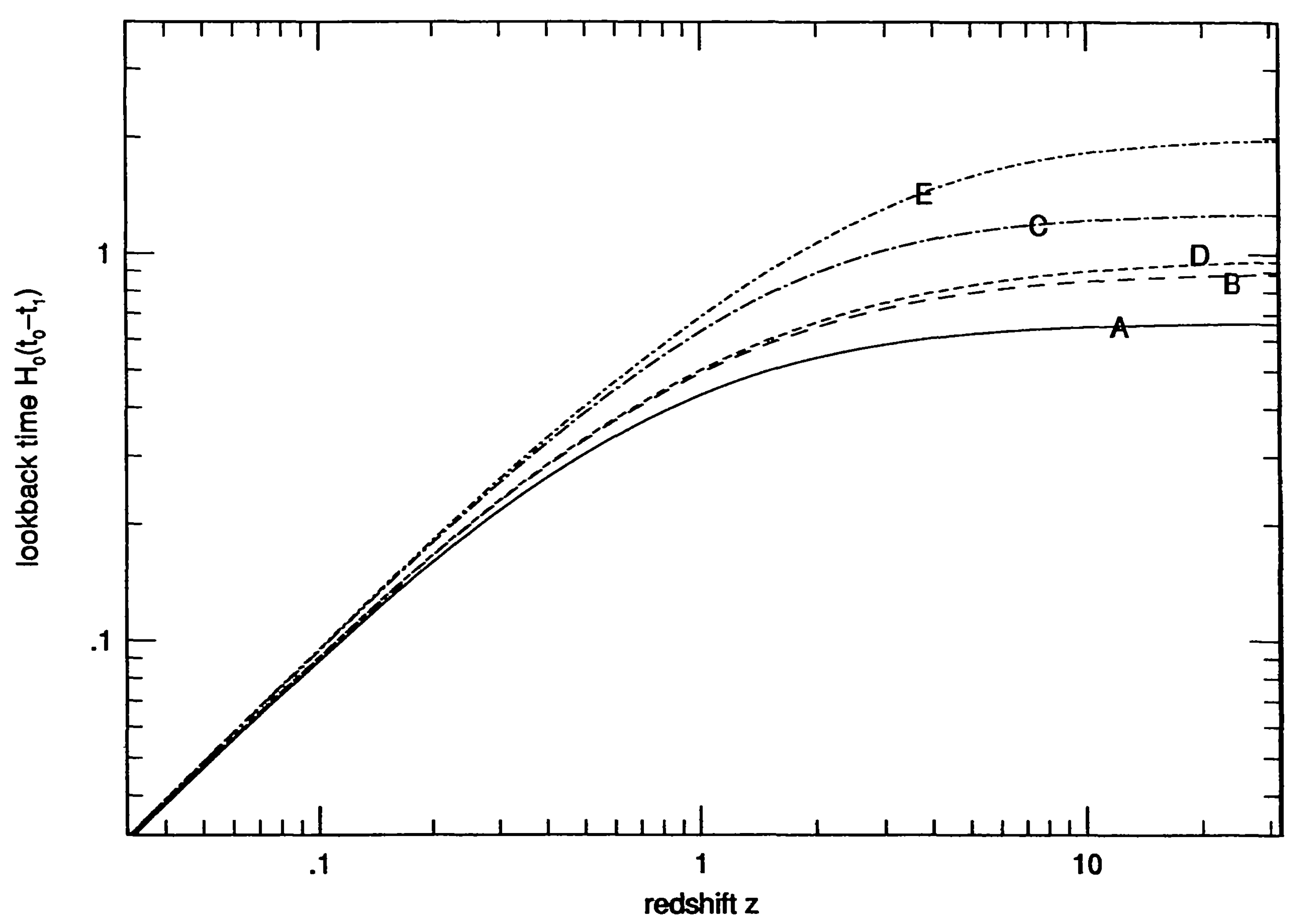
Start with $\left(\frac{da}{d\tau}\right)^2 = 1 + \Omega_M\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1)$

and change variables using $a = 1/(1+z)$ and $t = \tau H_0^{-1}$.

Integrate to find lookback time for a given redshift.

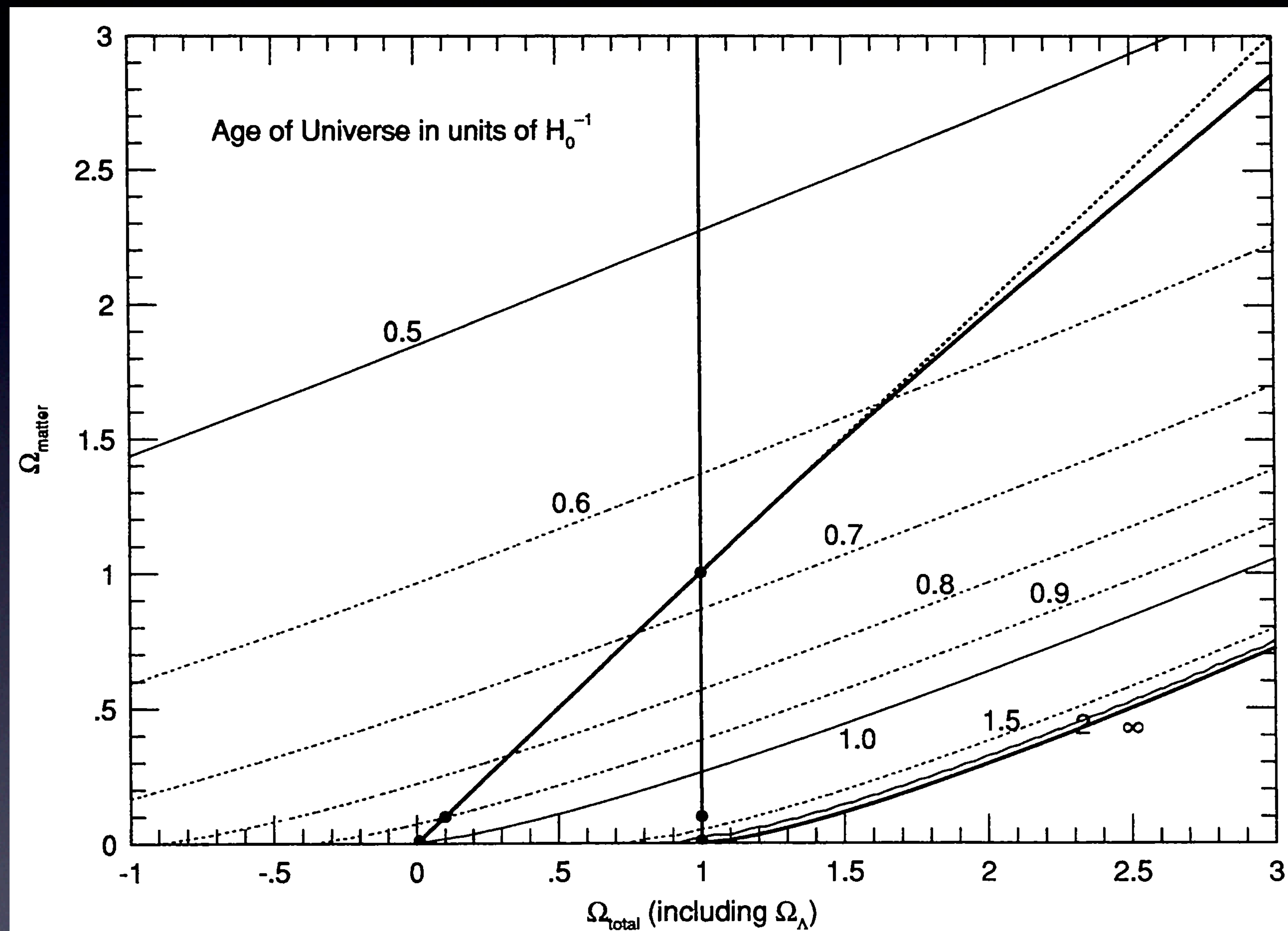
Time increases with increasing Ω_Λ, decreasing Ω_M.

Asymptotically approaches age of universe.



Model	Ω_{tot}	Ω_M	Ω_Λ	Description
A	1	1	0	flat, matter dominated, no Λ
B	0.1	0.1	0	open, plausible matter, no Λ
C	1	0.1	0.9	flat, Λ plus plausible matter
D	0.01	0.01	0	open, minimal matter, no Λ
E	1	0.01	0.99	flat, Λ plus minimal matter

Age of the Universe



Distance Measures

In order to determine the expansion history of the universe, we would like to measure the expansion factor, *R(t)*, and the coordinate radial distance *r(t)*.

But *R*, *r*, and *t* are not directly measurable.

Rather, we can measure things like:

- Redshift $z(t) = R_0/R(t) - 1$
- Angular Diameter Distance $d_A = D/\theta$
- Proper Motion Distance $d_m = u/\dot{\theta}$
- Luminosity Distance $d_L = \sqrt{L/4\pi F}$

The relations between the measurables and *R* and *r* are:

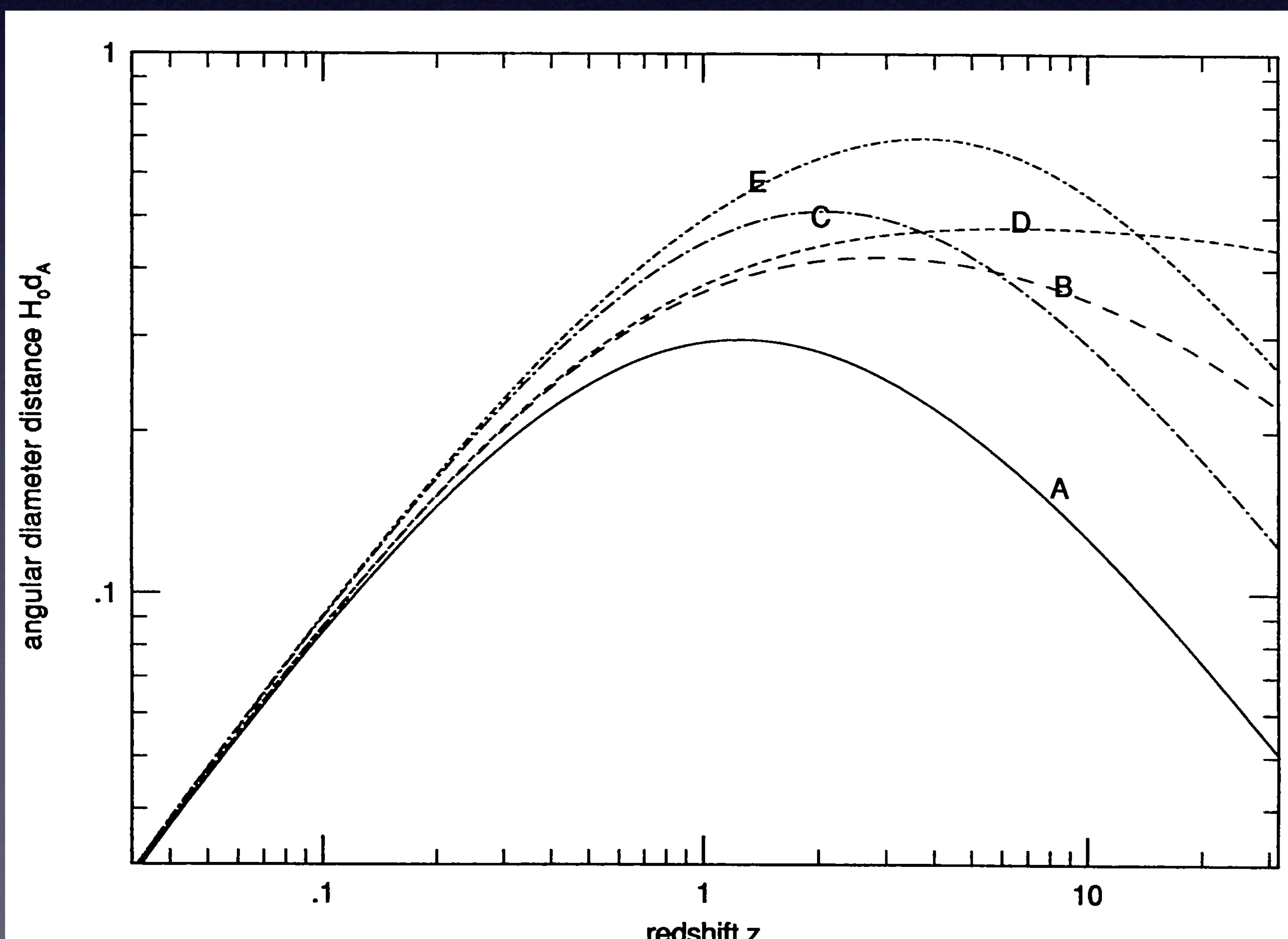
$$d_A = Rr, \quad d_m = R_0 r, \quad d_L = R_0^2 r/R$$

Distance Measures

We also have a relation between radial distance traveled and coordinate time elapsed for a light ray (null geodesic):

$$\frac{dr}{dt} = \left(-\frac{g_{00}}{g_{rr}} \right)^{1/2} = \frac{\sqrt{1-kr^2}}{R} \qquad \text{for the FRW metric.}$$

Integrating over *t* gives *r(t)* which can be combined with R_0 and *z* to give any of the above distance measures.



Model	Ω_{tot}	Ω_M	Ω_Λ	Description
A	1	1	0	flat, matter dominated, no Λ
B	0.1	0.1	0	open, plausible matter, no Λ
C	1	0.1	0.9	flat, Λ plus plausible matter
D	0.01	0.01	0	open, minimal matter, no Λ
E	1	0.01	0.99	flat, Λ plus minimal matter

Comoving Density

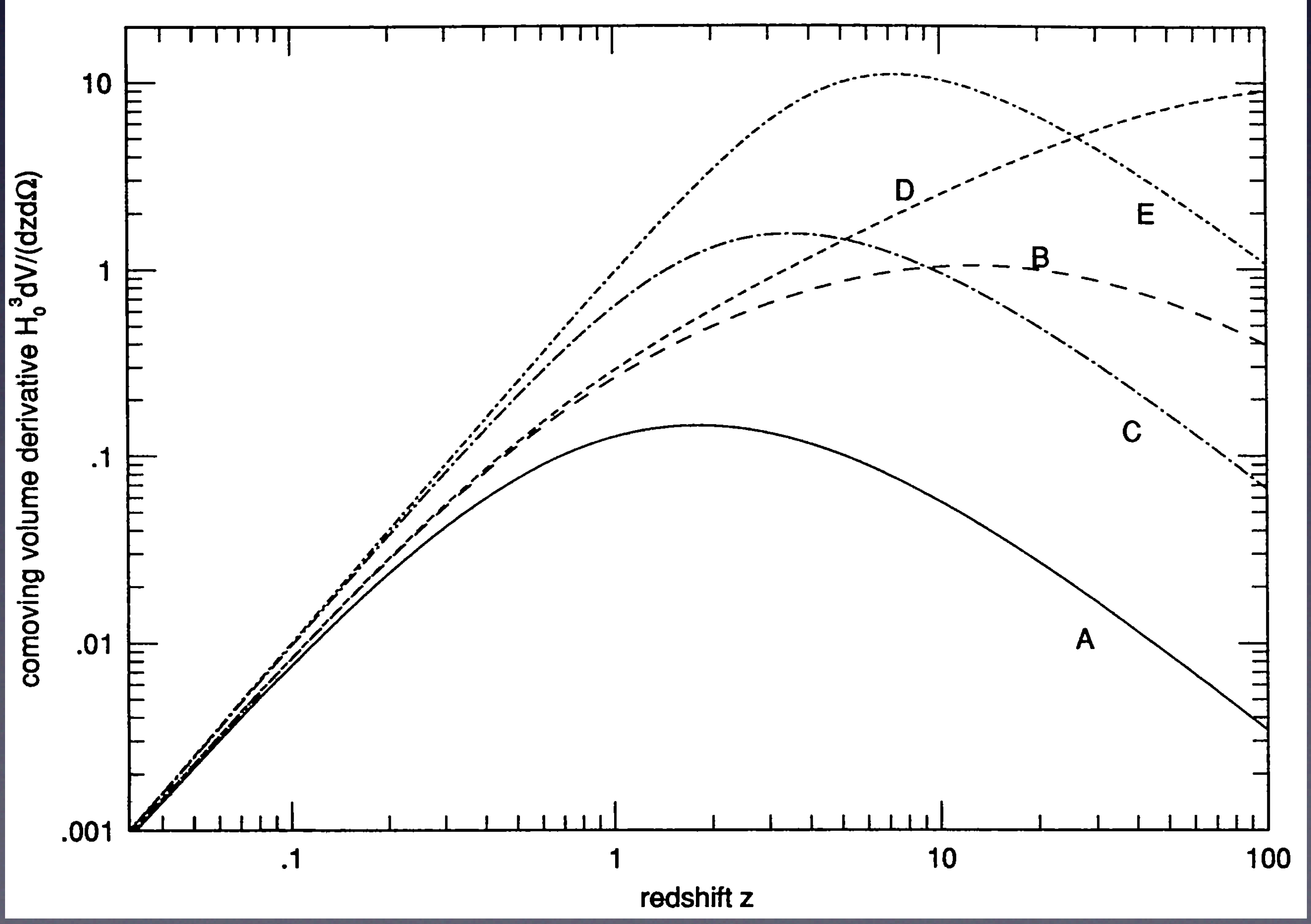
The volume element in the FRW metric is

$$dV = R_0^3 \frac{r^2}{(1-kr^2)^{1/2}} dr\, d\Omega = \frac{d_M^2}{(1+\Omega_k H_0^2 d_M^2)^{1/2}} d(d_M) d\Omega.$$

Direct dependence on curvature!

Any deviation from $V \propto d_m^3$ indicates non-flat geometry.

This is a powerful technique, but only if the evolution of the population is well understood.

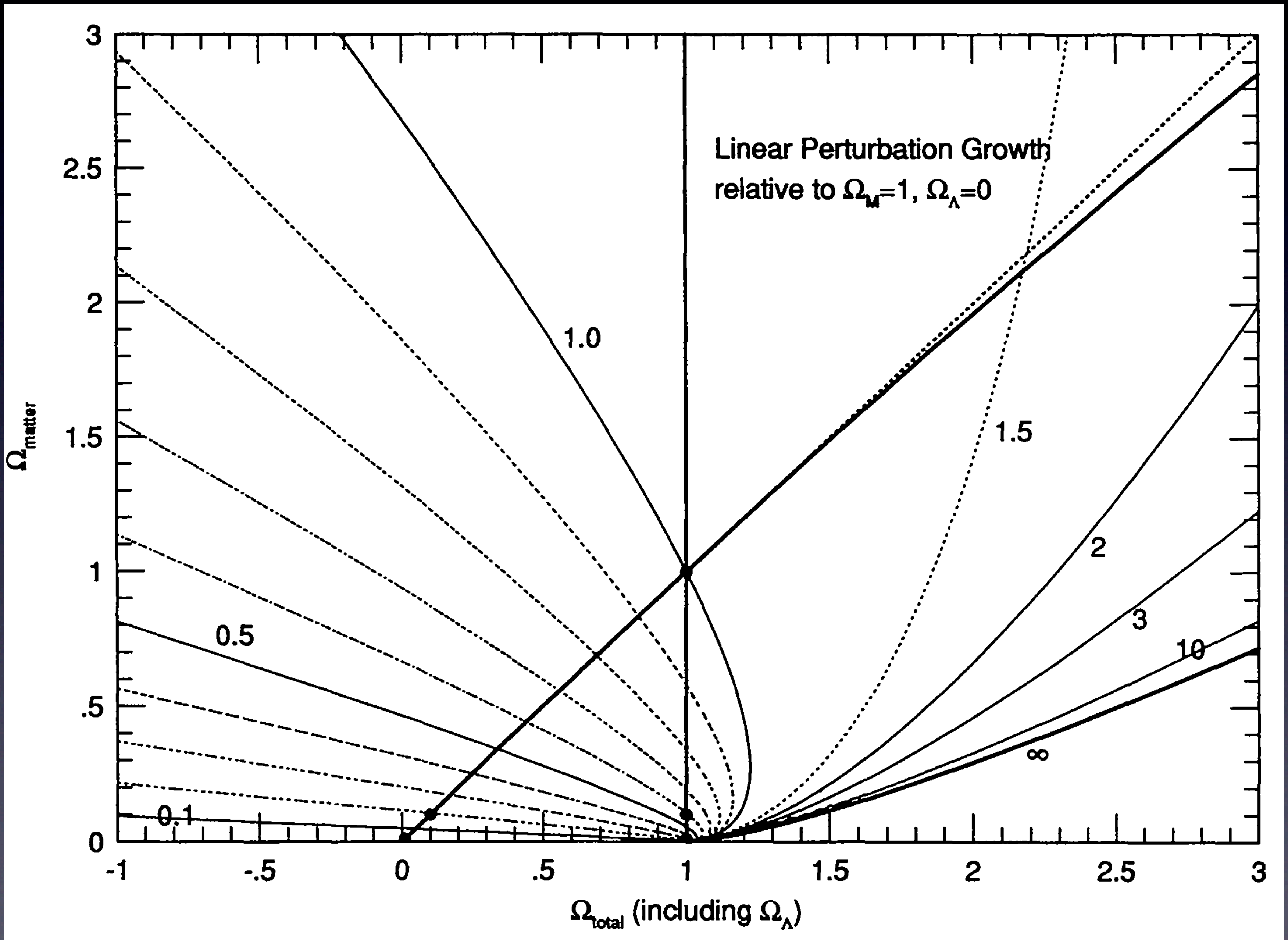


Model	Ω_{tot}	Ω_M	Ω_Λ	Description
A	1	1	0	flat, matter dominated, no Λ
B	0.1	0.1	0	open, plausible matter, no Λ
C	1	0.1	0.9	flat, Λ plus plausible matter
D	0.01	0.01	0	open, minimal matter, no Λ
E	1	0.01	0.99	flat, Λ plus minimal matter

Growth of Perturbations

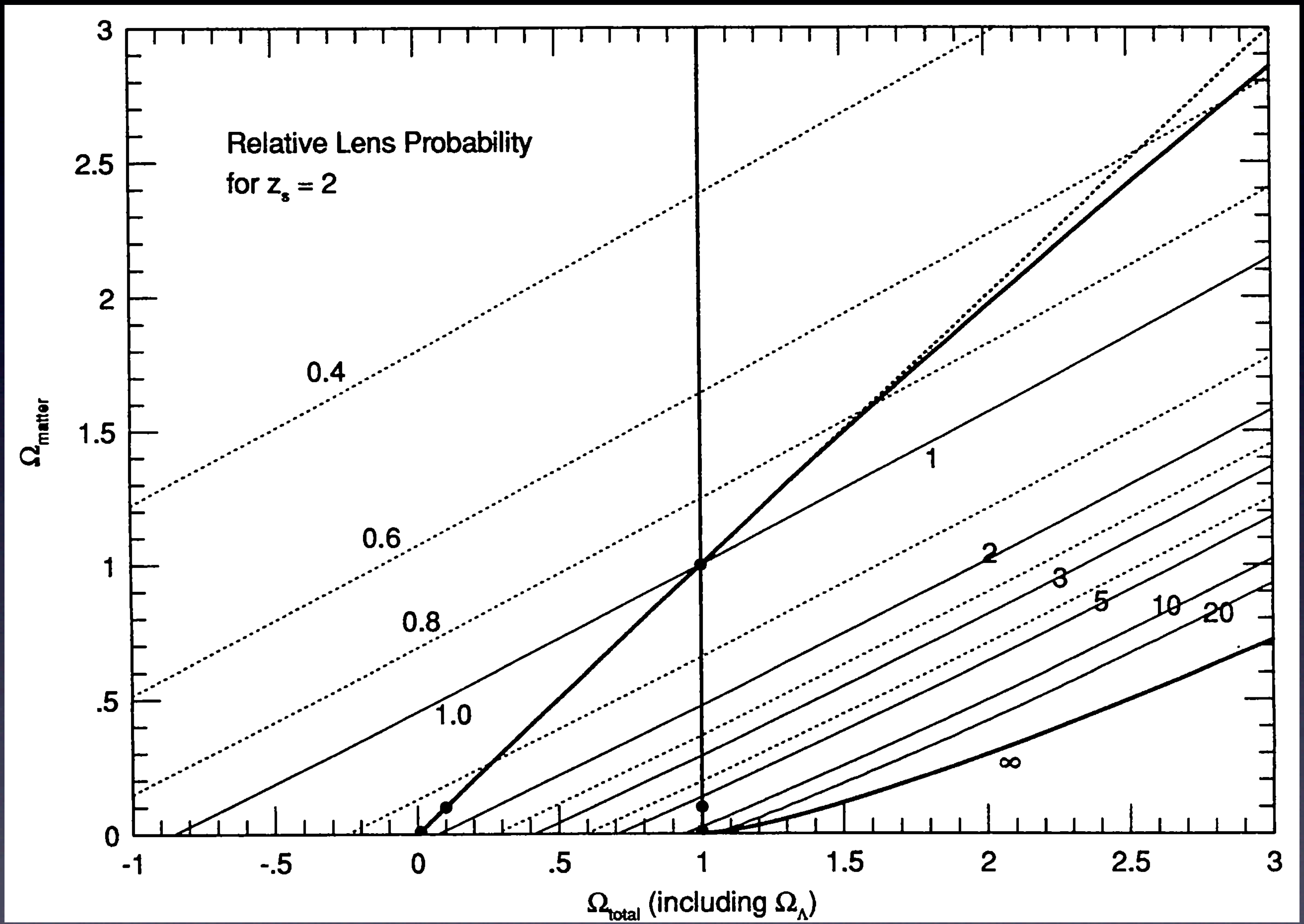
Linear Perturbation defined as a density enhancement $\delta \equiv \delta\rho/\rho.$

Plot of linear perturbation growth compared to $\Omega_M = 1,\ \Omega_\Lambda = 0.$



For fixed Ω_M, growth stops around $1+z = \Omega_M^{-1}$ for an open universe, but around $1+z = \Omega_M^{-1/3}$ for a flat universe.

Lensing Probabilities



Quantum cosmology: a possible solution

In the absence of a good quantum theory of gravity, start from the path integral formalism:

$$\Psi(\phi) \sim \int \mathcal{D}\mathrm{x}\; e^{iS[\mathrm{x}]/\hbar}$$

Now, consider the "states" to be 3D slices of the 4D spacetime.

Perform Wick rotation so that the metric becomes Euclidian and the action becomes imaginary.

$$S \to iS_E : \quad \Psi(\Sigma) \sim \int \mathcal{D}\mathrm{M}\; e^{-S_E[\mathrm{M}]/\hbar}$$

Now path integral will converge.

We can't do that integral, but sweep the details under a rug and define and effective action, Γ:

$$e^{-\Gamma[\mathrm{M}_c]/\hbar} \equiv \int \mathcal{D}\mathrm{M}\; e^{-S_E[\mathrm{M}]/\hbar}$$

where M_c is the "classical" space for which Γ is stationary. For large spaces, the leading term is just that of GR.

$$\Gamma = \frac{1}{16\pi G}\int d^4x\sqrt{g}(2\Lambda - R) + \ldots$$

Einstein's equation gives $R = 4\Lambda, \quad \int d^4x\sqrt{g} = 24\pi^2/\Lambda^2$

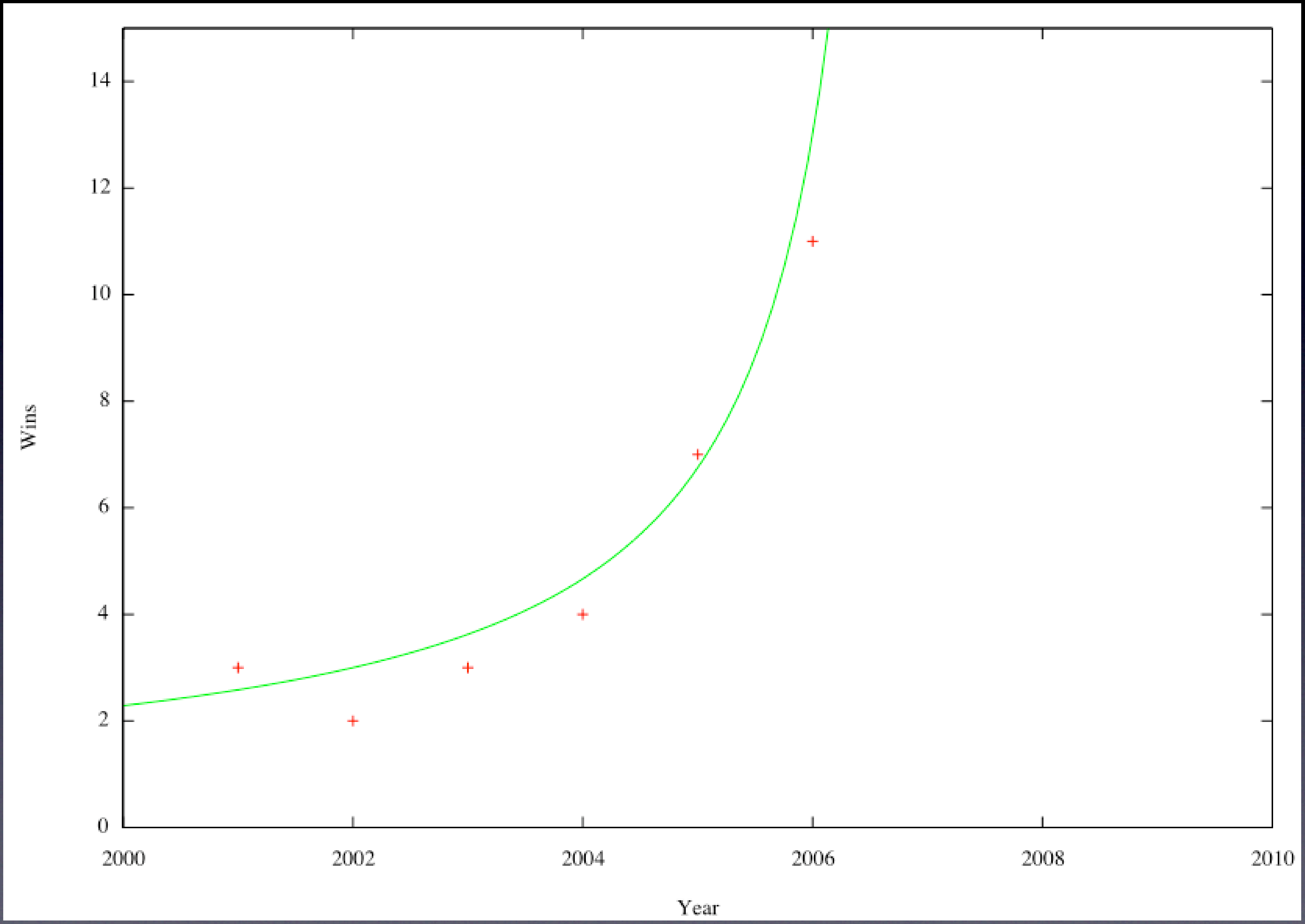
so that $\Psi \sim e^{3\pi/\hbar G\Lambda}$

which is infinitely peaked at $\Lambda = 0$

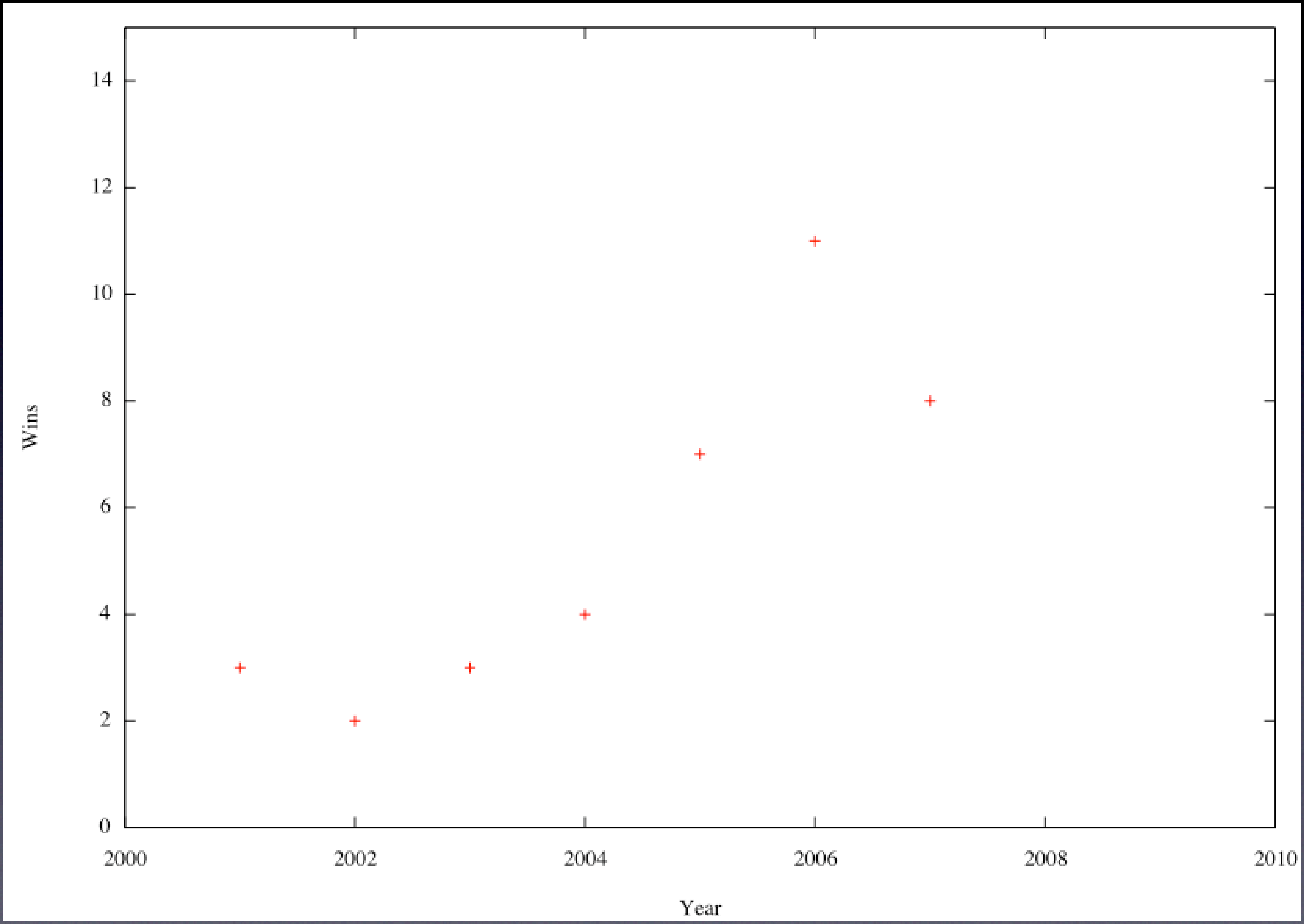
But Λ isn't really a free parameter...

- Λ-like scalar field...? (No other reason for it)
- Wormholes...? (Free in the action and produce a distribution of value for Λ)
- Other problems: Euclidian action not bounded below (!)
- Anthropic principle...?
- Other scalar fields...?
- Supersymmetry...?

Stadium Expansion



Stadium Expansion



Stadium Expansion