HW #2 (2025)

(1.) Consider the Cauchy process:

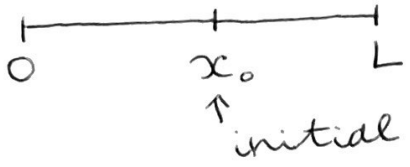
$$p(x, t+\Delta t \mid z, t) = \frac{\Delta t}{\pi[(x-z)^2 + \Delta t^2]} .$$

(a) Show that the Cauchy process does not satisfy the Lindeberg condition.

(b) Show that it satisfies the Chapman-Kolmogorov equation.
[Note that $\lim_{\Delta t \to 0} p(x, t+\Delta t \mid z, t) = \delta(x-z)$]

(c) Derive the jump rate $W(x \mid z, t)$ for the Cauchy process. Discuss its $x \to z$ limit.

(2.) Consider diffusion on a finite 1D interval:



unbiased RW

(a) For a particle located at x_0 at $t=0$, compute eventual exit probabilities (in the $t \to \infty$ limit)

$P_-(x_0)$ = prob. to exit at $x=0$

$P_+(x_0)$ = prob. to exit at $x=L$

what is $P_-(x_0) + P_+(x_0)$?

(b) Derive expressions for $t_+(x_0)$ and $t_-(x_0)$, the mean first-passage (MFPT) time for all paths that terminate at $x=L$ and $x=0$, respectively.

Is your answer consistent with the unconditional MFPT

$$t(x_0) = \frac{x_0(L-x_0)}{2D}$$ derived in class?