# Final Exam (2023)

[20 points]

(1.) Write down the 1D Fokker-Planck (FP) equation for a diffusive particle in the harmonic potential:

$$U(x) = \frac{m\omega^2 x^2}{2}, \text{ where}$$

$m$ is the particle mass and
$\omega$ is the oscillator frequency.

Solve the FP equation: find $P(x,t)$, the prob. to find the particle at coord. $x$ at time $t>0$ given that it was at $x_0$ at time $t=0$: $P(x,0) = \delta(x-x_0)$.

Hint: the solution is a gaussian, so one only needs the 1st & 2nd moments of $x$.

Discuss $t \to 0$ & $t \to \infty$ limits of $P(x,t)$.

[20 points]

(2.) Using transfer matrix techniques in a 1D Ising model with $H=0$:

$$\mathcal{H} = -J \sum_{i=0}^{N-1} S_i S_{i+1} \quad ; \quad S_i = \pm 1 \,, \ \forall i \,.$$

($J$: coupling constant)

and periodic boundary conditions, find the correlation length $\xi$ in the thermodynamic ($N \to \infty$) limit. Discuss the $T \to 0$ & $T \to \infty$ limits of $\xi$. What is the critical temperature $T_c$ in this system, and why?

[20 points]

3. Using the MaxEnt procedure, derive the microcanonical distribution of statistical mechanics.

Hint: the derivation is outlined in Pressé et al., Rev. Mod. Phys. 85 (2013).