

INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2026

Questions 2. (Due Weds, Oct 7)

1. In this question $c_i^\dagger$ and c_i are fermion creation and annihilation operators and the states are fermion states. Use the convention

$$|11111000\dots\rangle = c_5^\dagger c_4^\dagger c_3^\dagger c_2^\dagger c_1^\dagger |\text{vacuum}\rangle.$$

- (a) Evaluate $c_3^\dagger c_6 c_4 c_6^\dagger c_3 |111110000\dots\rangle$.
 - (b) Write $|1101001100\dots\rangle$ in terms of excitations about the “filled Fermi sea” $|1111100000\dots\rangle$. Interpret your answer in terms of electron and hole excitations.
 - (c) Find $\langle\psi|c_3^\dagger c_2^\dagger|\psi\rangle$ where $|\psi\rangle = A|100\rangle + B|111000\rangle$, $\hat{N} = \sum_i c_i^\dagger c_i$.
2. Write down the following operators in first and second-quantized notation
 - (a) The charge density $\rho_Q(x)$ of a quantum fluid of protons.
 - (b) The total magnetization $\vec{M}$ of a quantum fluid of electrons.
 - (c) The interaction energy $\hat{V}$ for a fluid of spin 1/2 fermions interacting via a contact interaction $U\delta^3(\vec{x})$.

3. (a) Consider two fermions, a_1 and a_2 . Show that the Bogoliubov transformation

$$c_1 = ua_1 + va_2^\dagger, \quad c_2^\dagger = -va_1 + ua_2^\dagger, \quad (1)$$

where u and v are real, preserves the canonical anti-commutation relations if $u^2 + v^2 = 1$.

- (b) Use this result to show that the Hamiltonian

$$H = \epsilon(a_1^\dagger a_1 - a_2 a_2^\dagger) + \Delta(a_1^\dagger a_2^\dagger + \text{H.c.}) \quad (2)$$

can be diagonalized in the form

$$H = \sqrt{\epsilon^2 + \Delta^2}(c_1^\dagger c_1 + c_2^\dagger c_2 - 1). \quad (3)$$

(Hint: work backwards from the answer.)

- (c) What is the ground-state energy for the system described by this Hamiltonian?

4. (i) Use the Jordan Wigner transformation to show that the one dimensional anisotropic XY model

$$H = -\sum_i [J_1 S_x(i) S_x(i+1) + J_2 S_y(i) S_y(i+1)] \quad (4)$$

can be written as

$$H = -\sum_i [t(d_{i+1}^\dagger d_i + \text{H.c.}) + \Delta(d_{i+1}^\dagger d_i^\dagger + \text{H.c.})] \quad (5)$$

where $t = \frac{1}{4}(J_1 + J_2)$ and $\Delta = \frac{1}{4}(J_2 - J_1)$.

- (ii) Calculate the excitation spectrum for this model and sketch your results. Comment specifically on the two cases $J_1 = J_2$ and $J_2 = 0$. (Hint: when you Fourier transform the pairing terms, since fermion operators anticommute, the resulting term must be odd in momentum, eliminating the even $\cos k$ part of the exponent, leaving just an odd $\sin k$ function.)