On Friday we started our consideration of **Lie** or **topological** groups, where the elements are parameterized by continuous parameters, such as the angle of a rotation. We saw that the set of group elements might have a piece connected to the identity, the **connected component**, as well as possible pieces not so connected. We saw how the connected component is described by a **manifold** of some finite dimension n, and that the group properties are largely determined by the commutators of the generators, which is to say the derivatives of the group elements with respect to the parameters, evaluated at the identity. These generators form a **Lie algebra**, whose basic bilinear operation is the commutator:

$$[L_i, L_j] = c_{ij}{}^k L_k,$$

where $c_{ij}{}^k$ are constants called the **structure constants**. They determine much of the group properties.

Any representation of the group leads to a representation of the Lie algebra, and then the group elements are given by exponentials of vectors in the Lie algebra.

We gave a formal definition of a Lie Algebra and interpreted the commutator of the Lie algebra vectors as a bilinear operation $\mathcal{L} \times \mathcal{L} \to \mathcal{L}$, and found the constraints on the structure constants due to the antisymmetry and the Jacobi identity.

We start today with some examples of Lie Algebras and begin to discuss the representations. We will discuss some familiar groups, in particular SO(3) and SU(2), discussing their manifolds and in particular their topology. We will then develop the **adjoint representation**, which for a Lie group plays a role somewhat similar to the regular representation of a finite group, but it is not the same thing. We will define the **Killing form** which is a matrix which will eventually provide a measure on the space near the identity, but more immediately will tell us properties of the algebra and group, in particular whether there is an abelian subspace of the algebra which generates a normal subgroup. We will be primarily interested in **semisimple** algebras for which there is no such **abelian invariant subalgebra**. We will also learn that semisimplicity and whether or not the group is compact is determined by whether the Killing form is nonsingular and positive definite.

We will then briefly discuss the Poincaré and Lorentz groups.

Reminders:

Homework 3 is due Thursday, Feb. 9.