Homework 2 has been posted and is due on Feb. 2. Solutions to homework 1 will be posted after class. They were delayed because one (unamed) one of you mistakenly thought it due today in class. Does that need discussion? The solutions are accessible from the homeworks page, but only if you log on with username physics618 and the password I will tell you now.

Review

Last time we briefly listed all the groups of order < 8, and then went on to the most important thing for physicists, **representations** of the group. An ℓ-dimensional representation is a homomorphism from the group into a set of $\ell \times \ell$ complex matrices. We distinguished reducible from irreducible representations according to whether the ℓ-dimensional vector space on which these matrices act could be broken into a direct sum of spaces, each closed under the group operations. We concentrated on finite-dimensional representations, and defined what it means for a representation to be faithful, trivial, reducible, or unitary, or to be the identity representation. We defined equivalence of two representations. We showed every representation of a finite group is equivalent to a unitary one. Then we proved Schur's first lemma:

> Any matrix which commutes with all the matrices of an irreducible representation of a finite or compact Lie group must be a multiple of the identity.

and I began working out an example, showing that the 6 dimensional space of the third level of a 3-D harmonic oscillator is not an irreducible representation of the rotation group. This was a bit of a stretch, as the rotation group is not a finite group, but it works anyway. [By the way, this observation, and the same for levels of the non-relativistic hydrogen atom, raises the question of whether there is a bigger symmetry than rotations, under which this *is* an irreducible representation.]

Today

Today we will begin with Schur's Second Lemma:

> Two irreducible reps are either equivalent or there is no matrix M other than zero for which $M\Gamma^i(A) = \Gamma^j(A)M$ for all $A \in G$.

Then we will turn to the **Great Orthogonality Theorem** which tells us that the set of inequivalent finite dimensional representations provides a complete and independent set of basis functions for the vector space of functions on the group. This will tell us that $\sum_i \ell_i^2 = g$, where the sum is over all

inequivalent irreducible representions of dimension ℓ_i. This is very powerful in helping us find them.

Finally we will get to **characters** of the represention, the traces of the representation matrices. These are maps from the group into $\mathbb{C}$ which are unaffected by similarity transformations, and are therefore functions from the conjugacy classes into $\mathbb{C}$. Again we will use completeness and independence to show that the number of inequivalent irreducible representions is equal to the number of conjugacy classes, and thus how many terms there are in the $\sum_i \ell_i^2$.

Perhaps we will have time to work some examples.