Physics 618 Fifteenth Lecture March 10, 2017

Last time we examined some of the representations of SU(3) by choosing the q^i (and thus the highest weights $\vec{\mu}_{\rm max} = \sum q^i \vec{\mu}_i$, in terms of the fundamental weights), and working down with lowering operators $E_{-\vec{\alpha}^i}$ for the simple roots. We found the $q = (1, 0)$ is the quark representations, and $q = (0, 1)$ the antiquark. We considered the $\underline{\mathbf{15}}$ or $q = (2, 1)$ representation and shortened our efforts by noting that there are Weyl reflections in hyperplanes perpendicular to every root under which the weight diagrams are symmetric.

Then we turned to tensor methods, observing that any representation can be built up by tensor products of the fundamental represention, followed by reduction to irreducible multiplets. Here we took the basis vectors of the tensor product as the direct product of those of the fundamental components, *e.g.* ${e^{ij}}_k = e^i \times e^j \times e_k$, and a general vector is expressed by its coefficients ${v_{ij}}^k$. We saw that the Lie algebra generators act as sum of actions on each index, and observed that this preserves any permutation symmetry properties of the upstairs indices, and also of the downstairs ones, so a tensor product of more than one $\underline{\mathbf{3}}$ will be reducible into parts symmetric or antisymmetric, or some more complicated permutation symmetry.

Today

We will work a few simple examples. We will note that tracing in one upper and one lower index extracts a reduced piece. We will then extract an arbitrary (q^1, q^2) irreducible representation of SU(3) from traceless tensors totally symmetric in upper indices and also in the lower indices, and find the dimension of these.

This is particularly effective for SU(3). But for higher N this is less helpful, so we observe that we didn't really need the lower indices if we are willing to consider fancier permutation symmetries. This will work for SU(N) as well, and we will be able to find all the SU(N) representations using only the *defining* representation. We will see how permutation symmetry plays a crucial role in reducing the tensor products. For SU(3) we do not need fancy permutations, but for the higher SU(N) we will need to understand representations of the permutation group in detail. This will involve finding all the irreducible representations of S_n, which we already know are P(n) in number, where $P(n)$ is the number of partitions of n. We will also define the **group algebra**.

For a thorough exploration of the permutations you could get a copy of Irene Verona Schensted "A Course on the Application of Group Theory to Quantum Mechanics", NEO Press 1976, which I did years ago for $6, but the best I found last year was $50 for a used copy from Amazon!

Enjoy the vacation. See you March 21.