Physics 618 Fourteenth Lecture March 3, 2017

Last time we finished up our finding of all simple compact Lie groups. Having previously shown the Dynkin diagram to contain at most only one multiple bond **or** one 3-legged branch point, we proceeded to find limits on the single-bond chains that could be attached. Thus we found the full list of simple compact Lie groups included 4 infinite series, $A_n = SU(n+1)$, $B_n = SO(2n+1)$, $C_n = Sp(2n)$ and $D_n = SO(2n)$, and five exceptional groups E_6, E_7, E_8, F_4 and G_2.

While the Dynkin diagrams depend only on the Cartan matrix $A_{ji} = 2\dfrac{\vec{\alpha}_j \cdot \vec{\alpha}_i}{\alpha_i^2}$ between simple roots, all the roots $\vec{\mu}$ of the algebra can be found by recursive raising by simple roots (and adding in their negatives to the list). We discussed a recipe which I encased in wine bottle that automates this process, giving all the positive roots in a Hasse diagram.

Then we turned to the question of finding the irreducible representations of our Lie algebra/group. We defined the unique highest weight vector $\vec{\mu}_{\max}$, and, as that cannot be raised by any simple root, the set of corresponding q^i's expresses the same information as the weights, and are always nonnegative integers.

We will define the **fundamental weights** $\vec{\mu}^{\,i}$ of the algebra to be the maximum weights giving one q, $q^i = 1$, with the rest zero, so $2\dfrac{\vec{\alpha}_i \cdot \vec{\mu}^j}{\alpha_i^2} = \delta_{ij}$, and the representation with that weight, D^i, is a called a **fundamental representation** (of which there are m).

Today

Today we will examine some of the representations of SU(3) by choosing the q^i (and thus the highest weights), and working down with lowering operators. We will first discuss the antiquark and quark representations, and discuss how flavor SU(3) is discussed in terms of isospin, charge, strangeness and (strong) hypercharge. We will also look at the octet and at a 15 dimensional representation of no direct physical importance but one which will display some new features. We will do this in an ad-hoc way, though we could use a procedure analogous to our bottles working downwards. We will also discuss Weyl reflection symmetry as well as complex conjugation.

Any representation can be built up by tensor products of the fundamental represention, followed by reduction to irreducible multiplets. This is particularly effective for SU(N), and even more so for SU(3). We will see how permutation symmetry plays a crucial role in reducing the tensor products. For SU(3) we do not need fancy permutations, but for the higher SU(N) we will need to understand representations of the permutation group in detail. This, however, will have to wait until after the exam.

Reminders:

The midterm exam is next Tuesday, March 7. This will cover the material from the beginning of the term through Dynkin diagrams and the construction of the full algebra (the bottle graphs). It will not cover the current chapter on finding the general representations of the algebras.

The exam will be closed book, but you are allowed two sheets of $8\frac{1}{2} \times 11$ inch handwritten notes.