HW #5 (2026)

1. $\boxed{40\text{ pt}}$ [Domain walls in Landau-Ginzburg theory]

Consider a 1D system described by the Landau-Ginzburg free energy:

$$F[m] = \int_{-\infty}^{\infty} dx \left[\frac{c}{2}\left(\frac{dm}{dx}\right)^2 + \frac{a}{2} m^2 + \frac{b}{4} m^4 \right],$$

where $m = m(x)$ is magnetization, $b > 0$, $c > 0$, and $a = \tilde{a}(T - T_c)$, $\tilde{a} > 0$. Note that $a < 0$ if $T < T_c$.

(a) Find equilibrium values of m for $T < T_c$, in the uniform case: $m = \text{const}(x)$.

(b) Find the differential equation for $m(x)$ that minimizes $F[m]$ at equilibrium (hint: use Euler-Lagrange approach)

(c) Find the domain-wall solution of the above equation, with
$m(x \to -\infty) = -m_0$, $m(x \to +\infty) = m_0$.

Show that $m(x) = m_0 \tanh\left(\frac{x}{\sqrt{2}\,\xi}\right)$, and express ξ and m_0 in terms of a, b, c.

Discuss the physical meaning of ξ and its behavior as $T \to T_c$.

(2.) $\boxed{20 \text{ pt}}$ [The Chinese philosophers problem]

Consider N philosophers sitting at a circular table. Each philosopher can either meditate (state -1) or eat (state $+1$). To eat, each philosopher needs 2 chopsticks. Chopsticks are shared with the 2 neighboring philosophers, one on the left and one on the right. Thus, if a philosopher is eating, his left & right neighbors cannot eat at the same time.

(a) Show that the problem is equivalent to N spins with periodic BCs:
$\sigma_i \in \{-1, +1\}$, $i = 1, \dots, N$

(b) Evaluate the total number of allowed configurations M_N for $N = 2, 3, 4, 5$

(c) Show that $M_N = M_{N-1} + M_{N-2}$
<u>Hint</u>: consider $M_N(\sigma)$ = # acceptable configurations with $\sigma_N = \sigma$, where $\sigma \in \{-1, 1\}$

(d) Show that the problem reduces to counting the minimal energy states of a 1D Ising model given by:

$$\mathcal{H} = \sum_{i=1}^{N} \left[-J\sigma_i\sigma_{i+1} - \frac{h}{2}(\sigma_i + \sigma_{i+1}) \right],$$

with $\sigma_{i+N} = \sigma_i$ (periodic BCs),

$J = -1$, $h = -2$.

Find the corresponding ground-state energy $E_0(N)$.

(e) Show that

$$M_N = \lim_{\beta\to\infty} e^{\beta E_0(N)} Z_N, \quad \text{where}$$

$$Z_N = \sum_{\{\sigma\}} e^{-\beta\mathcal{H}(\{\sigma\})}.$$

Find Z_N using the transfer matrix technique (OK to use results from the lecture notes) and evaluate M_N in this way. Does it match the expressions obtained above?