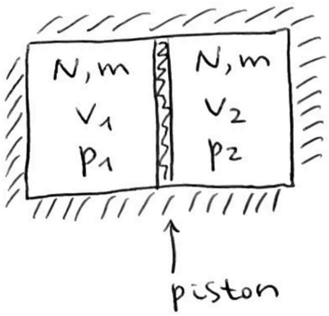
HW #4 (2026)

① **20 pt** A thermally insulated cylinder is separated into two compartments by a frictionless piston:



Both compartments have the same total number of particles N with mass m. Ideal Fermi gases form in each compartment. Particles in compartment 1 have spin S_1;
— 〃 — — 〃 — ——— 〃 ——— 2 have spin S_2.

Compute the volume ratio $\frac{V_2}{V_1}$ at thermal equilibrium with:

(a) $T = 0$

(b) $T = \infty$

(2.) 20 pt Consider 2D ideal Bose gas.

(a) Find the grand-canonical partition function $\Sigma(\mu, V, T)$ in terms of

$$g_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}.$$

(b) Find pressure p in this system.

(c) Find chemical potential μ in terms of T and $n = \frac{N}{A}$, the # part. per unit area.

(N: total # part.; A: area $= L^2$)

(3.) [20 pt] Consider quantum spinless particles with the dispersion relation: $\epsilon(\vec{k}) = c|\vec{k}|$. The system is contained in a cubic d-dim box: $V = L^d$.

(a) Write down the expression for the grand canonical partition function (no need to evaluate the integral)

(b) Write down the expression for $\langle N \rangle$, the average number of particles. (again, leave it as an integral)

(c) Using the expression for $\langle N \rangle$ obtained above, argue that Bose-Einstein condensation only takes place if $d \geq 2$. What happens if $d = 1$?