HW #2 solutions (2026)

1. 2.19

Recall that
$$\begin{cases} C_V = T\left(\frac{\partial S}{\partial T}\right)_V, & C_p = T\left(\frac{\partial S}{\partial T}\right)_p \\ K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S, & K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \end{cases}$$

Assume that $n = \text{const}$.

Consider $\frac{K_S}{K_T} = \left(\frac{\partial V}{\partial p}\right)_S \Big/ \left(\frac{\partial V}{\partial p}\right)_T \stackrel{\text{①}}{=}$

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

$$\stackrel{\text{①}}{=} \left(\frac{\partial V}{\partial S}\right)_p \left(\frac{\partial S}{\partial p}\right)_V \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = \left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial T}{\partial S}\right)_p \boxed{=}$$

chain rule

$\boxed{=}\ \frac{C_V}{C_p}$, as desired.

Moreover, recall that
$$C_p - C_V = \underbrace{-T\left(\frac{\partial p}{\partial V}\right)_T}_{\substack{<0 \\ \text{by stability}}} \underbrace{\left[\left(\frac{\partial V}{\partial T}\right)_p\right]^2}_{>0} > 0$$

$$\boxed{C_p > C_V}$$

But then $\frac{K_T}{K_S} = \frac{C_p}{C_V} > 1 \Rightarrow \boxed{K_T > K_S}$

It is easier to compress gases isothermically than adiabatically.

2. 2.20

(a) Rubber band heating up when it is stretched adiabatically implies that

$$\left(\frac{\partial T}{\partial L}\right)_{S,n} > 0 .$$

Since $\left(\frac{\partial T}{\partial f}\right)_{S,n} = \left(\frac{\partial T}{\partial L}\right)_{S,n} \underbrace{\left(\frac{\partial L}{\partial f}\right)_{S,n}}_{>0 \text{ by stability}}$ $\Rightarrow$

$\Rightarrow$ $\left(\frac{\partial T}{\partial f}\right)_{S,n} > 0$ as well.

Now, consider

$$\left(\frac{\partial L}{\partial T}\right)_{f,n} = \left(\frac{\partial L}{\partial S}\right)_{f,n} \left(\frac{\partial S}{\partial T}\right)_{f,n} \;=$$

$$dH = d(E - fL) \underset{n=\text{const}}{=} TdS - Ldf \quad \text{implies}$$

$$\left(\frac{\partial T}{\partial f}\right)_{S,n} = -\left(\frac{\partial L}{\partial S}\right)_{f,n}$$

$$= -\underbrace{\left(\frac{\partial T}{\partial f}\right)_{S,n}}_{>0} \underbrace{\left(\frac{\partial S}{\partial T}\right)_{f,n}}_{>0 \text{ by stability}} < 0 .$$

Thus, the rubber band stretches ($\Delta L > 0$) when cooled ($\Delta T < 0$).

(b) Consider

$$\left(\frac{\partial S}{\partial T}\right)_f = \left(\frac{\partial S}{\partial T}\right)_L \underbrace{\left(\frac{\partial T}{\partial T}\right)_f}_{=1} + \left(\frac{\partial S}{\partial L}\right)_T \left(\frac{\partial L}{\partial T}\right)_f \text{, or}$$

n = const everywhere

$$dS = \left(\frac{\partial S}{\partial T}\right)_L dT + \left(\frac{\partial S}{\partial L}\right)_T dL$$

$S = S(T, L)$

$$\left(\frac{\partial S}{\partial T}\right)_f - \left(\frac{\partial S}{\partial T}\right)_L = \left(\frac{\partial S}{\partial L}\right)_T \left[\underbrace{-\left(\frac{\partial f}{\partial T}\right)_L \left(\frac{\partial L}{\partial f}\right)_T}_{=\left(\frac{\partial L}{\partial T}\right)_f} \right] =$$

$$= \underbrace{\left(\frac{\partial S}{\partial L}\right)_T^2}_{>0} \underbrace{\left(\frac{\partial L}{\partial f}\right)_T}_{>0 \text{ by stability}} > 0.$$

$dA = d(E - TS) = -S\,dT + f\,dL$ implies that

$$\left(\frac{\partial f}{\partial T}\right)_L = -\left(\frac{\partial S}{\partial L}\right)_T$$

Thus, $C_f - C_L > 0$, or

$$\frac{1}{C_L} > \frac{1}{C_f}.$$

If $\Delta Q = T\Delta S$ is the same, the $L =$ const rubber band will undergo the larger change in T: $\Delta T_L > \Delta T_f$.

③ 2.26

$$\begin{cases} \text{phase } \alpha: & \beta p = a + b\beta\mu , \\ \text{Phase } \gamma: & \beta p = c + d(\beta\mu)^2 . \end{cases}$$

phase equilibrium: $\beta^{(\alpha)} = \beta^{(\gamma)} = \beta$,

$$p^{(\alpha)} = p^{(\gamma)} = p, \quad \mu^{(\alpha)} = \mu^{(\gamma)} = \mu .$$

Thus, $\quad a + b\beta\mu = c + d(\beta\mu)^2$, or

$$\beta\mu = \frac{b \pm \sqrt{b^2 - 4d(c-a)}}{2d} .$$

Since $c < a$, $d > 0$, $\sqrt{b^2 - 4d(c-a)} > b$ and only the positive root is physical.

Now, use GD equation:

$$d\mu = -\underbrace{s}_{= \frac{S}{n}} dT + \underbrace{v}_{= \frac{V}{n}} dp \Rightarrow \frac{1}{v} = \rho = \left(\frac{\partial p}{\partial \mu}\right)_T = \left(\frac{\partial(\beta p)}{\partial(\beta\mu)}\right)_\beta .$$

Then $\begin{cases} \rho^{(\alpha)} = b , \\ \rho^{(\gamma)} = 2d\beta\mu . \end{cases}$

Finally, $\rho^{(\gamma)} - \rho^{(\alpha)} = \left(b + \sqrt{b^2 - 4d(c-a)}\right) - b =$
$= \sqrt{b^2 + 4d(a-c)}$, and

$$\underbrace{\beta p}_{\text{@ transition}} = a + \frac{b}{2d}\left(b + \sqrt{b^2 + 4d(a-c)}\right) .$$