HW # 1 solutions (2026)

1. $S = k\,(N V E)^{1/3}$, yielding

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{k}{3}\left(\frac{N V}{E^2}\right)^{1/3} .$$

at equilibrium,

$$\frac{N^A V^A}{(E^A)^2} = \frac{N^B V^B}{(E^B)^2} ,$$ where

$$E^A + E^B = E = \text{const} .$$

Thus, $$(E - E^A)^2 N^A V^A = (E^A)^2 N^B V^B ,$$

$$\left(\frac{E^A}{E - E_A}\right)^2 = \frac{N^A V^A}{N^B V^B} ,$$ or

$$\begin{cases} E^A = \dfrac{E_0}{1 + \sqrt{\dfrac{N^B V^B}{N^A V^A}}} , \\[2ex] E^B = \dfrac{E_0}{1 + \sqrt{\dfrac{N^A V^A}{N^B V^B}}} . \end{cases}$$

2.

(a) at $p = \text{const}$,

$$dS = C_p \frac{dT}{T}.$$

Then $dE = \underbrace{C_p\,dT}_{T\,dS} - p\,dV \underset{\uparrow}{=} (C_p - Nk_B)\,dT$

$p\,dV = Nk_B\,dT$ for ideal gas @ $p = \text{const}$

On the other hand, $E = E(T)$ for an ideal gas:

$$dE = \underbrace{C_V}_{\left(\frac{\partial E}{\partial T}\right)_V} dT$$

← $dE = dQ$ since $V = \text{const}$, no mechanical work is done on the system

can also show formally that $\left(\frac{\partial E}{\partial V}\right)_T = 0$

Thus, $C_V = C_p - Nk_B \Rightarrow \underline{\underline{C_p - C_V = Nk_B}}$.

(b) Now we consider the adiabatic process: $dS = 0$.

We have $dE = C_V\,dT = -p\,dV$.

The equation of state now gives

$$dT = \frac{1}{Nk_B} d(pV) = \frac{1}{C_p - C_V}(p\,dV + V\,dp), \text{ or}$$

$$p\,dV + V\,dp = -\frac{C_p - C_V}{C_V} p\,dV,$$

$$V\,dp = -\frac{C_p}{C_V} p\,dV.$$

Finally, $\left(\frac{\partial p}{\partial V}\right)_S = -\frac{C_p}{C_V}\frac{p}{V} = -\gamma \frac{p}{V}$, as desired

(c) Use the equation of state again:

$$V = \frac{N k_B T}{p} \quad \text{gives}$$

$$dV = N k_B \left(\frac{dT}{p} - \frac{T}{p^2} dp \right).$$

Using $V dp = -\gamma p dV$, we obtain:

$$\gamma \cancel{N} k_B T \frac{dp}{p} = \quad , \text{ or}$$

$$\gamma \cancel{N} k_B dT + \frac{\cancel{N} k_B T}{p} dp$$

$$\frac{dp}{p}(1-\gamma) + \gamma \frac{dT}{T} = 0.$$

Since $\gamma = \text{const}$,

$$d \log \left(p T^{\gamma/(1-\gamma)} \right) = 0, \text{ or}$$

$$p T^{\frac{\gamma}{1-\gamma}} = \text{const} \quad \text{for an adiabatic process}$$

3. Chandler 1.15

We start with $dE = TdS - pdV + \mu dn$,

which yields $dS = \frac{1}{T}dE + \frac{p}{T}dV - \frac{\mu}{T}dn$.

Then $S = S(E, V, n)$, as expected.

Consider $\Phi = S - \frac{1}{T}E$:

$$d\Phi = \underbrace{dS}_{\frac{1}{T}dE + \frac{p}{T}dV - \frac{\mu}{T}dn} - \frac{1}{T}dE - E\,d\left(\frac{1}{T}\right) = -E\,d\left(\frac{1}{T}\right) + \frac{p}{T}dV - \frac{\mu}{T}dn$$

We see that $\Phi = \Phi\left(\frac{1}{T}, V, n\right)$, as requested.

↑ Massieu potential

Next, consider $\Psi = S - \frac{1}{T}E + \frac{\mu}{T}n$:

$$d\Psi = \underbrace{dS}_{\frac{1}{T}dE + \frac{p}{T}dV - \frac{\mu}{T}dn} - \frac{1}{T}dE - E\,d\left(\frac{1}{T}\right) + \frac{\mu}{T}dn + n\,d\left(\frac{\mu}{T}\right) =$$

$$= -E\,d\left(\frac{1}{T}\right) + \frac{p}{T}dV + n\,d\left(\frac{\mu}{T}\right).$$

Clearly, $\Psi = \Psi\left(\frac{1}{T}, V, \frac{\mu}{T}\right)$.

Now, let's use $\beta = \frac{1}{T}$ $(k_B = 1)$

Since $V = \text{const}$ in all terms of the formula we want, we can just omit V-dependence and restore it at the end.

Note that

$$n = \left(\frac{\partial \Psi}{\partial(\beta\mu)}\right)_\beta \underset{\substack{\uparrow \\ \Psi = \Psi(\beta,\, \beta\mu)}}{=} n(\beta, \beta\mu).$$

Then

$$\left(\frac{\partial n}{\partial \beta}\right)_{\beta\mu} \left(\frac{\partial \beta}{\partial(\beta\mu)}\right)_n \left(\frac{\partial(\beta\mu)}{\partial n}\right)_\beta = -1, \text{ or}$$

$$-\left(\frac{\partial(\beta\mu)}{\partial\beta}\right)_n \left(\frac{\partial n}{\partial(\beta\mu)}\right)_\beta = \left(\frac{\partial n}{\partial\beta}\right)_{\beta\mu}.$$

Now, $E = E(S, V, n) \Rightarrow E(S, n)$ can be transformed to $E(\beta, n)$.

Then

$$dE = \left(\frac{\partial E}{\partial \beta}\right)_n d\beta + \left(\frac{\partial E}{\partial n}\right)_\beta dn, \text{ or}$$

$$\left(\frac{\partial E}{\partial\beta}\right)_{\beta\mu} = \left(\frac{\partial E}{\partial\beta}\right)_n \underbrace{\left(\frac{\partial\beta}{\partial\beta}\right)_{\beta\mu}}_{=1} + \left(\frac{\partial E}{\partial n}\right)_\beta \left(\frac{\partial n}{\partial \beta}\right)_{\beta\mu} =$$

$$= \left(\frac{\partial E}{\partial n}\right)_\beta \left(\frac{\partial n}{\partial\beta}\right)_{\beta\mu} + \left(\frac{\partial E}{\partial\beta}\right)_n \underset{\substack{\uparrow \\ \text{restore} \\ V}}{=} -\left(\frac{\partial E}{\partial n}\right)_{\beta,V} \left(\frac{\partial n}{\partial(\beta\mu)}\right)_{\beta,V} \left(\frac{\partial(\beta\mu)}{\partial\beta}\right)_{n,V} + \left(\frac{\partial E}{\partial\beta}\right)_{n,V}, \text{ as desired}$$

$\boxed{\text{QED}}$

4. Chandler 1.16

Recall that $C_\ell = T\left(\frac{\partial S}{\partial T}\right)_\ell$, where

both C_ℓ & S are per unit mass.

Then $\left(\frac{\partial C_\ell}{\partial \ell}\right)_T = \left(\frac{\partial}{\partial \ell}\left[T\left(\frac{\partial S}{\partial T}\right)_\ell\right]\right)_T =$

$$= T\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial \ell}\right)_T\right)_\ell .$$

$\frac{\partial}{\partial \ell}$ is taken at $T = \text{const}$

Next, we use

$$dE = TdS + fdL + \mu dn ,$$ which gives

$$d(\underbrace{E-TS}_{A}) = -SdT + fdL + \mu dn .$$

we obtain a Maxwell relation:

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L ,$$ or

$$\left(\frac{\partial S}{\partial \ell}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_\ell .$$

Hence $\left(\frac{\partial C_\ell}{\partial \ell}\right)_T = -T\left(\frac{\partial^2 f}{\partial T^2}\right)_\ell = 0$, in fact.

$f = \frac{\ell T}{\theta}$