Consider van der Waals gas described by the following equation of state:
\[ P = \frac{RT}{V-b} - \frac{a}{V^2}, \]
where \( a \) and \( b \) are constants.

Show that \( C_V \) (the heat capacity at constant \( V \)) is a function of \( T \) alone for the vdw gas.

Note: there is no need to find the specific form of this function.
Show that the entropy per photon in blackbody radiation is given by

\[ S = \frac{4k}{8} \frac{\sum_{n=1}^{\infty} n^{-4}}{\sum_{n=1}^{\infty} n^{-3}} \text{ in 3 spatial dimensions.} \]

Note: recall that

\[ \zeta(s) = \frac{1}{\Gamma(s)} \int_{0}^{\infty} \frac{x^{s-1}}{e^x - 1} \, dx = \sum_{n=1}^{\infty} \frac{1}{n^s}, \]

where \( \zeta(s) \) is the Riemann zeta function and \( \Gamma(s) = \int_{0}^{\infty} x^{s-1} e^{-x} \, dx \) is the Gamma function.