Midterm (2024)

(1.) 25 pt

Consider van der Waals (vdW) gas described by the following equation of state:

$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$

(p: pressure), where a & b are constants.

Show that C_v (the heat capacity at constant V) is a function of T alone for the vdW gas.

Note: there is no need to find the specific form of this function.

(2.) 25 pt

Show that the entropy per photon in blackbody radiation is given by

$$S = 4k_B \frac{\sum_{n=1}^{\infty} n^{-4}}{\sum_{n=1}^{\infty} n^{-3}}$$

in 3 spatial dimensions.

Note: recall that

$$\underbrace{\zeta(s)}_{\text{Riemann zeta function}} = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1}\, dx = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

where $\Gamma(s) = \int_0^\infty x^{s-1} e^{-x}\, dx$ is the gamma function