1. A cylinder is separated into two compartments by a free-sliding piston:

Two ideal Fermi gases are placed into the compartments: compartment 1 has particles with mass $M$, spin $S = \frac{1}{2}$, while compartment 2 has particles with mass $M$, spin $S = \frac{3}{2}$. Both gases have the same total number of particles $N$.

Find the ratio of the two volumes, $\frac{V_2}{V_1}$, at equilibrium with $T=0$ and $T=\infty$. 
2. Consider ideal Bose gas.

(a) Find $\log \sum (z, V, T)$ as a 1D integral. 

\[ \text{grand-canonical partition function} \]

(b) Find $\frac{P}{k_B T}$ as a function of $\beta$.

\[ g_2(z) = \sum_{k=1}^{\infty} \frac{2^k}{k^2} \quad \text{and} \quad \lambda = \sqrt{\frac{2\pi \hbar^2}{mk_B T}}. \]

Pressure

Hint: expand the integral in (a) in powers of $z$.

(c) Find the average number of particles per unit area as a function of $z$ and $T$; insert the equation to find $z = z(n, T)$. 

\[ \frac{N}{L^2} \]