HOMEWORK 2 - TIME-DEPENDENT PERTURBATION

Problem 1.

1.1 A harmonic oscillator potential is subjected to the perturbation λbx^2 in the time between 0 to T. Obtain the selection rules for the transition from the initial state ϕ_i to ϕ_f in time T and the transition probabilities for the possible transitions.

1.2 If the perturbation added to a harmonic oscillator potential is λbx^3 find the selection rules and the transition probabilities for the allowed transitions.

Comment on the selection rules for λbx^n

Hint: To solve, express the perturbation in terms of annihilation and creation operators.

Problem 2.

At time $t = 0$ the infinite height potential $V(x) = 0$ for $0 < x < L$ and ∞ otherwise is perturbed by the additional term of the form $V_p(x) = V_0$ for $L/4 < x < 3L/4$ and 0 otherwise. The perturbation is switched-off at $t = T$. The system is initially in the ground state ϕ_1. What is the probability of finding it in the state ϕ_3 after the time $t=T$?

Problem 3.

Assume that an adiabatic perturbation of the form $H^{(1)} = W(x)e^{\alpha t}$ is turned on slowly from $t = -\infty$. Obtain the expression for second-order transition amplitude. Also write the time-independent wave function up to second-order correction.

Problem 4.

A one-dimensional harmonic oscillator has its spring constant k suddenly reduced by a factor of 1/2. The oscillator is initially in its ground state. Find the probability for the oscillator to remain in the ground state after the perturbation.