HOMEWORK 1 – PHYS 502

PERTURBATION METHODS I

Problem 1.

A harmonic oscillator potential is perturbed by the term $\lambda b x^2$. Calculate the first-order and second-order corrections to the energy eigenvalues.

Hint: Express the Hamiltonian in terms of creation and annihilation operators.

Problem 2.

If the Hamiltonian of a particle in a box of length L is subjected to a uniform electric field given by $H^{(1)} = -eEx$ calculate the first-order correction to the energy eigenvalues.

Hint:

The eigenvalues and the eigenfunctions of the unperturbed system are

$$E_n^{(0)} = \frac{\hbar^2 \pi^2 n^2}{4mL^2}\,, \quad n = 1, 2, \cdots, \quad \phi_n^{(0)} = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\,.$$

Problem 3.

An electron in the potential $V(x) = \begin{cases} -a/x, & x > 0 \\ \infty, & x \leq 0 \end{cases}$ is in its ground state characterized by the eigenfunction $\phi_0 = Nx\mathrm{e}^{-\beta x}$. The particle is subjected to an applied electric field E_e in the x-direction. Calculate the first-order correction to ground state energy eigenvalue.

Hints:

First, calculate β. Substituting the solution in the Schrödinger equation

$$-\frac{\hbar^2}{2m}\psi_{xx} - \frac{a}{x}\psi = E\psi$$

Next, from the normalization condition $1 = N^2 \int_0^\infty x^2 \mathrm{e}^{-2\beta x}\,\mathrm{d}x$ we find $N = 2\beta^{3/2}$.