PHYSICS 502 - ADVANCED QUANTUM - final exam

Return your solutions as a pdf file on **Saturday May 11 by 10 am latest** at jak.chakhalian@rutgers.edu

Problem 1.

Calculate the first and second-orders corrections to the energy eigenvalues of a linear harmonic oscillator with the cubic term $-\lambda\mu x^3$ added to the potential. Discuss the condition for the validity of the approximation.

Problem 2.

The Hamiltonian of a perturbed system is $H = \begin{pmatrix} 1 & 2\epsilon & 0 \\ 2\epsilon & 2+\epsilon & 3\epsilon \\ 0 & 3\epsilon & 3+2\epsilon \end{pmatrix}$ where $\epsilon \ll 1$. Workout the first-order eigenvalues and eigenvectors using the perturbation theory.

Problem 3.

Evaluate the transition amplitude upto the second-order for the constant perturbation $V(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \geq 0. \end{cases}$

Problem 4.

A particle in a box potential of width L is perturbed by the term $V_0 \sin(\pi x/L)$ during the time 0 to T. Compute the probability for the transition from the ground state ϕ_1 to the excited state ϕ_3 in time T.

Problem 5.

A one-dimensional harmonic oscillator has its spring constant k suddenly reduced by a factor of $1/2$. The oscillator is initially in its ground state. Find the probability for the oscillator to remain in the ground state after the perturbation.

Problem 6.

Using the WKB quantization rule find the eigenvalues of the quartic anharmonic oscillator with the Hamiltonian $H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \lambda x^4$.

Problem 7.

For a one-dimensional box of dimension L with eigenfunction ϕ show that $\langle E \rangle = (\hbar^2/2m)\int_0^L |\mathrm{d}\phi/\mathrm{d}x|^2\, \mathrm{d}x$. Using this relation estimate the ground state energy for a particle in the one-dimensional box with trial eigenfunction

$\phi(x) = \begin{cases} x/(\beta L), & 0 \le x \le \beta L \\ (L-x)/((1-\beta)L), & \beta L \le x \le L. \end{cases}$ Taking β as the variational parameter compare it with the exact result.

Problem 7.

Estimate the ground state of the infinite-well (one-dimensional box) problem defined by

$$V = \begin{cases} 0, & \text{for } |x| < L \\ \infty, & \text{for } |x| > L, \end{cases}$$

using the trial eigenfunction $\phi = |L|^\alpha - |x|^\alpha$ with α the trial parameter and compare it with the exact energy value.

Problem 8.

Calculate the differential cross-section for a central Gaussian potential $V(r) = (V_0/\sqrt{4\pi})\mathrm{e}^{-r^2/4a^2}$ under Born approximation.