The θ term

In particle physics and condensed matter physics

Anna Hallin

601:SSP, Rutgers 2017

Preliminaries

- The language of particle physics is Quantum field theory
  - Particles = quantized excitations of fields

Preliminaries

- The language of particle physics is Quantum field theory
  - Particles = quantized excitations of fields
- The Standard Model of particle physics
  - Matter fields: electrons, quarks... $\psi(x)$
  - Gauge fields: photons, gluons... $A_\mu(x)$ with field strength $F_{\mu\nu}$
  - Higgs field: $\phi(x)$

Preliminaries

- The language of particle physics is Quantum field theory
  - Particles = quantized excitations of fields
- The Standard Model of particle physics
  - Matter fields: electrons, quarks... $\psi(x)$
  - Gauge fields: photons, gluons... $A_\mu(x)$ with field strength $F_{\mu\nu}$
  - Higgs field: $\phi(x)$
- The Lagrangian controls evolution and interactions of fields
  - $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \bar{\psi}_i y_{ij}\psi_j\phi + |D_\mu\phi|^2 - V(\phi)$

The θ term

There is another term we can add to the Lagrangian that respects Lorentz invariance and gauge invariance:

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} \widetilde{F}^a_{\mu\nu} F^{a,\mu\nu} \tag{1}$$

where

$$\widetilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{a,\rho\sigma} \tag{2}$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \underbrace{ig[A^b_\mu, A^c_\nu]}_{\text{0 if abelian}} \tag{3}$$

The θ term

There is another term we can add to the Lagrangian that respects Lorentz invariance and gauge invariance:

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} \widetilde{F}^a_{\mu\nu} F^{a,\mu\nu} \tag{1}$$

where

$$\widetilde{F}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{a,\rho\sigma} \tag{2}$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \underbrace{ig[A^b_\mu, A^c_\nu]}_{\text{0 if abelian}} \tag{3}$$

- In particle physics, the the θ term will only affect QCD, the physics of the strong interaction. A^a_μ are gluons.
- In condensed matter physics, the θ term will affect the electrodynamics. A_μ is the photon.

The strong CP problem

- In particle physics we are *forced* to include $\mathcal{L}_\theta$.
- However, it violates CP:
    - C = charge conjugation (flip particle $\leftrightarrow$ antiparticle)
    - P = parity ($x_i \leftrightarrow -x_i$)
    - CP: $\mathcal{L}_\theta \to -\mathcal{L}_\theta$

The strong CP problem

- In particle physics we are *forced* to include $\mathcal{L}_\theta$.
- However, it violates CP:
  - C = charge conjugation (flip particle $\leftrightarrow$ antiparticle)
  - P = parity $(x_i \leftrightarrow -x_i)$
  - CP: $\mathcal{L}_\theta \rightarrow -\mathcal{L}_\theta$
- The amount of CP violation in QCD is heavily constrained by experimental results: $\theta \leq 10^{-10}$
- The question of why θ is so small, or even zero, is called *the strong CP problem*.

The axion

- Peccei and Quinn (1977) introduced a mechanism to solve the strong CP problem: a broken symmetry resulting in the axion field $a(x)$. The axion is a pseudo ($m_a \neq 0$) Goldstone boson.

The axion

- Peccei and Quinn (1977) introduced a mechanism to solve the strong CP problem: a broken symmetry resulting in the axion field $a(x)$. The axion is a pseudo ($m_a \neq 0$) Goldstone boson.
- The potential for the axion field is

$$V(a) = \left(\theta + \frac{a(x)}{f_a}\right) \frac{g_s^2}{32\pi^2} \widetilde{F}^{\mu\nu,a} F_{\mu\nu,a} \tag{4}$$

- As the axion falls into its minimum, $a = -f_a\theta$, the θ term is effectively erased and the strong CP problem is solved.

The axion

- Peccei and Quinn (1977) introduced a mechanism to solve the strong CP problem: a broken symmetry resulting in the axion field $a(x)$. The axion is a pseudo ($m_a \neq 0$) Goldstone boson.
- The potential for the axion field is

$$V(a) = \left(\theta + \frac{a(x)}{f_a}\right) \frac{g_s^2}{32\pi^2} \widetilde{F}^{\mu\nu,a} F_{\mu\nu,a} \tag{4}$$

- As the axion falls into its minimum, $a = -f_a\theta$, the θ term is effectively erased and the strong CP problem is solved.
- So far, no experiment has detected an axion, but there is still a lot of parameter space to cover. If some version of axion exists, it could also be a candidate for dark matter.

The θ term in condensed matter physics

- In condensed matter physics, we are not forced to introduce $\mathcal{L}_\theta$. But interesting things happen if we do!

Topological insulators from a particle physics perspective

- Let's begin with electrodynamics, and add fermions:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi + m \bar{\psi} \psi \quad (5)$$

Topological insulators from a particle physics perspective

- Let's begin with electrodynamics, and add fermions:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi + m\bar{\psi}\psi \tag{5}$$

- We will write the fermions as chiral Weyl fermions, χ and η. Then the mass term above becomes

$$m\bar{\psi}\psi \rightarrow m\chi\eta + m^*\chi^*\eta^* \tag{6}$$

Note that the mass parameter is allowed to be complex!

Topological insulators from a particle physics perspective

- Write the complex mass in terms of its magnitude and phase:

$$m\chi\eta + m^*\chi^*\eta^* = |m|e^{i\theta}\chi\eta + |m|e^{-i\theta}\chi^*\eta^* \tag{7}$$

Topological insulators from a particle physics perspective

- Write the complex mass in terms of its magnitude and phase:

$$m\chi\eta + m^*\chi^*\eta^* = |m|e^{i\theta}\chi\eta + |m|e^{-i\theta}\chi^*\eta^* \quad (7)$$

- Applying the time reversal operator leads to complex conjugation of numbers: $T : c \to c^*$, so if we want our theory to be invariant under time reversal we must demand the mass parameter to be real.
- Hence there are only two possible values for θ: 0 or π.

Topological insulators from a particle physics perspective

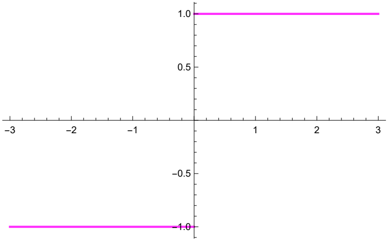
- Write the complex mass in terms of its magnitude and phase:

$$m\chi\eta + m^*\chi^*\eta^* = |m|e^{i\theta}\chi\eta + |m|e^{-i\theta}\chi^*\eta^* \tag{7}$$

- Applying the time reversal operator leads to complex conjugation of numbers: $T : c \to c^*$, so if we want our theory to be invariant under time reversal we must demand the mass parameter to be real.
- Hence there are only two possible values for θ: 0 or π.
- Assign the value $\theta = \pi$ to a topological insulator, and $\theta = 0$ to a regular insulator. Then $m_{topo} < 0$ and $m_{reg} > 0$.

Topological insulators from a particle physics perspective

- On the boundary between a topological insulator and a regular one, the mass parameter must cross zero:



- This theory of a time-reversal invariant topological insulator seems to predict massless fermions living on its surface! So far so good.

Topological insulators from a particle physics perspective

- Assume we are at energies $E \ll m$, and promote the parameter θ to a field, $\theta(x)$.
- It turns out that the effective theory contains a term

$$\mathcal{L}_\theta \propto \theta \widetilde{F}_{\mu\nu} F^{\mu\nu} \tag{8}$$

This is the θ term!

Topological insulators from a particle physics perspective

- Assume we are at energies $E \ll m$, and promote the parameter θ to a field, $\theta(x)$.
- It turns out that the effective theory contains a term

$$\mathcal{L}_\theta \propto \theta \widetilde{F}_{\mu\nu} F^{\mu\nu} \tag{8}$$

  This is the θ term!
- Under time reversal, $\widetilde{F}_{\mu\nu} F^{\mu\nu} \rightarrow -\widetilde{F}_{\mu\nu} F^{\mu\nu}$. If we want time-reversal invariance, we'd better have $\theta \rightarrow -\theta$, so that $T : \mathcal{L}_\theta = \mathcal{L}_\theta$.
- This is fine, since both $\pm\pi$ give us the same result for the mass parameter in the original theory: $m = |m| e^{i\theta} = |m| e^{\pm i\pi} = -|m|$.

Topological insulators from a particle physics perspective

- Assume we are at energies $E \ll m$, and promote the parameter θ to a field, $\theta(x)$.
- It turns out that the effective theory contains a term

$$\mathcal{L}_\theta \propto \theta \widetilde{F}_{\mu\nu} F^{\mu\nu} \tag{8}$$

  This is the θ term!
- Under time reversal, $\widetilde{F}_{\mu\nu} F^{\mu\nu} \to -\widetilde{F}_{\mu\nu} F^{\mu\nu}$. If we want time-reversal invariance, we'd better have $\theta \to -\theta$, so that $T : \mathcal{L}_\theta = \mathcal{L}_\theta$.
- This is fine, since both $\pm\pi$ give us the same result for the mass parameter in the original theory: $m = |m|e^{i\theta} = |m|e^{\pm i\pi} = -|m|$.
- We thus see that a time-reversal invariant θ term can be modelled as appearing in an effective theory of QED and fermions, in the limit that $m \to \infty$.

Axion electrodynamics

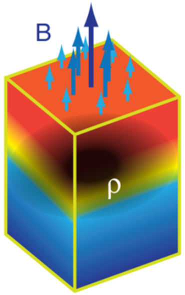
- Including the θ term corresponds to a change in Maxwell's equations.

Axion electrodynamics

- Including the θ term corresponds to a change in Maxwell's equations.
- If we write $\mathcal{L}_\theta = \kappa\theta \mathbf{E} \cdot \mathbf{B}$, the modified equations are
  - $\nabla \cdot \mathbf{E} = \rho - \kappa \nabla\theta \cdot \mathbf{B}$
  - $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$
  - $\nabla \cdot \mathbf{B} = 0$
  - $\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t + \mathbf{j} + \kappa\left(\partial\theta/\partial t \mathbf{B} + \nabla\theta \times \mathbf{E}\right)$
- Note: only gradients of θ occur here. If θ is a constant, Maxwell's equations are not modified.

Predictions: 1. Electric charge

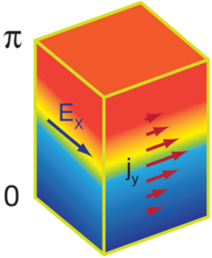
- The axion term in $\nabla \cdot \mathbf{E} = \rho - \kappa \nabla\theta \cdot \mathbf{B}$ tells us that an applied magnetic field will give a contribution to the electric charge density, provided $\nabla\theta \neq 0$.
- We would thus expect that an applied perpendicular magnetic field will cause an electric charge to build up on the interface between a topological insulator and a regular insulator ($\Delta\theta = \pi$).
- Interestingly, the charge per flux quantum is $e(n + 1/2)$. Hence each flux quantum gets attached to it a *fractional* charge.



M. Franz, "Viewpoint: High-energy physics in a new guise", Physics **1**, 36 (2008).

Predictions: 2. Fractional quantum Hall effect

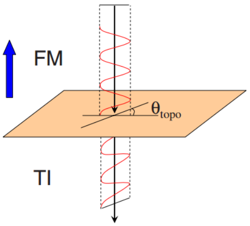
- From the axion term in $\nabla \times \mathbf{B} = \partial\mathbf{E}/\partial t + \mathbf{j} + \kappa\left(\partial\theta/\partial t\mathbf{B} + \nabla\theta \times \mathbf{E}\right)$, we would expect another effect.
- If we apply an electric field perpendicular to the interface, eg in the x-direction, a current will be generated in the y-direction.
- Note: in contrast to the regular Hall effect, this does not require an external magnetic field.
- It turns out that the Hall conductance $\sigma_H \propto (n + 1/2)$, which means that we are seeing a type of *fractional* quantum Hall effect.



M. Franz, "Viewpoint: High-energy physics in a new guise", Physics **1**, 36 (2008).

Experiment

- Faraday rotation can be a way to look for the effect of axion electrodynamics.
- As linearly polarized light passes through an interface between a ferromagnet and a topological insulator, the polarization will become rotated due to the boundary effects. This is a rotation that is in addition to the one caused by the ferromagnetic material.



X.-L. Qi, T. L. Hughes,S.-C.Zhang, "Topological field theory of time-reversal invariant insulators", Phys. Rev. B **78**, 195424 (2008)

Experiment

REPORTS

TOPOLOGICAL MATTER

Quantized Faraday and Kerr rotation and axion electrodynamics of a 3D topological insulator

Liang Wu,[1*]† M. Salehi,[2] N. Koirala,[3] J. Moon,[3] S. Oh,[3] N. P. Armitage[1*]

Topological insulators have been proposed to be best characterized as bulk magnetoelectric materials that show response functions quantized in terms of fundamental physical constants. Here, we lower the chemical potential of three-dimensional (3D) Bi_2Se_3 films to ~30 meV above the Dirac point and probe their low-energy electrodynamic response in the presence of magnetic fields with high-precision time-domain terahertz polarimetry. For fields higher than 5 tesla, we observed quantized Faraday and Kerr rotations, whereas the dc transport is still semiclassical. A nontrivial Berry's phase offset to these values gives evidence for axion electrodynamics and the topological magnetoelectric effect. The time structure used in these measurements allows a direct measure of the fine-structure constant based on a topological invariant of a solid-state system.

L. Wu, M. Salehi, N. Koirala, J. Moon, S. Oh, N. P. Armitage, "Quantized Faraday and Kerr rotation and axion electrodynamics of a 3D topological insulator". Science **354**, 1124 (2016).

Summary - what did we learn?

- We learned that in particle physics, the θ term gives rise to the strong CP problem. A proposed solution to this problem is the axion field.
- We saw how one can model a time-reversal invariant topological insulator using QED and massive fermions.
- The low-energy limit of this theory gives us the θ term, which leads to axion electrodynamics. This predicted the following effects:
  - Fractional charge attached to magnetic flux
  - Fractional (half-integer) quantum Hall effect
- Finally, we saw that researchers at Rutgers found evidence for axion electrodynamics through investigating Faraday and Kerr rotations.

Thank you!