September 22nd, 2017

Topological Phase Transition
(a.k.a. Kosterlitz-Thouless Transition)

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2016 Nobel Prize in Physics

Awarded for the theoretical discovery of topological phase transitions and topological phases of matter.

Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2

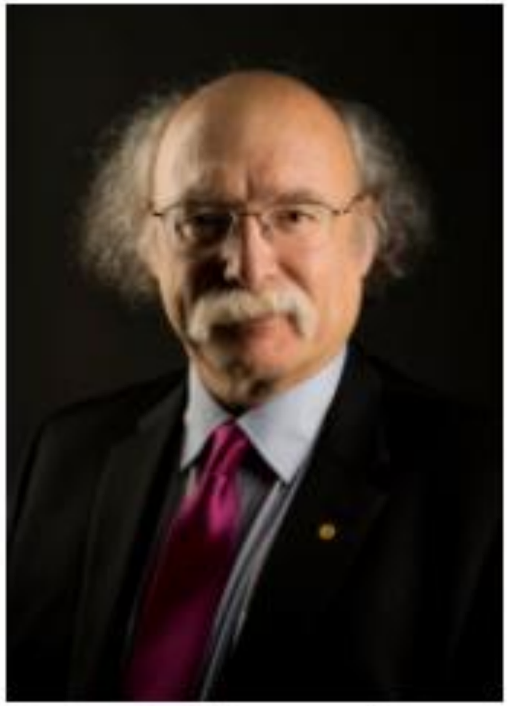

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Photo: A. Mahmoud
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What is topology?

- A branch of mathematics describing properties that are stable and only change in **integer** steps.

- Energy required to change the topology

- E.g. Topologically speaking: topological invariant is the number of holes

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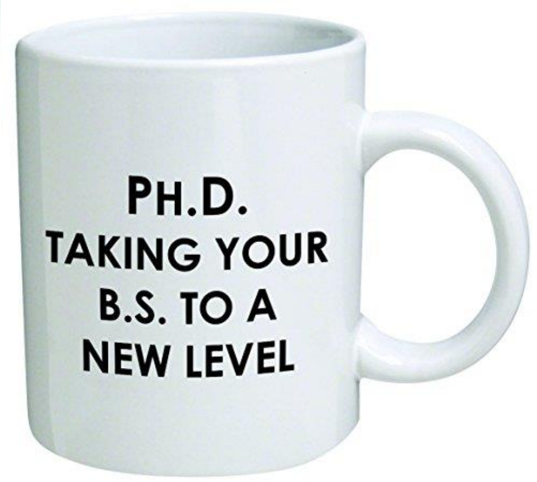
(n=1)



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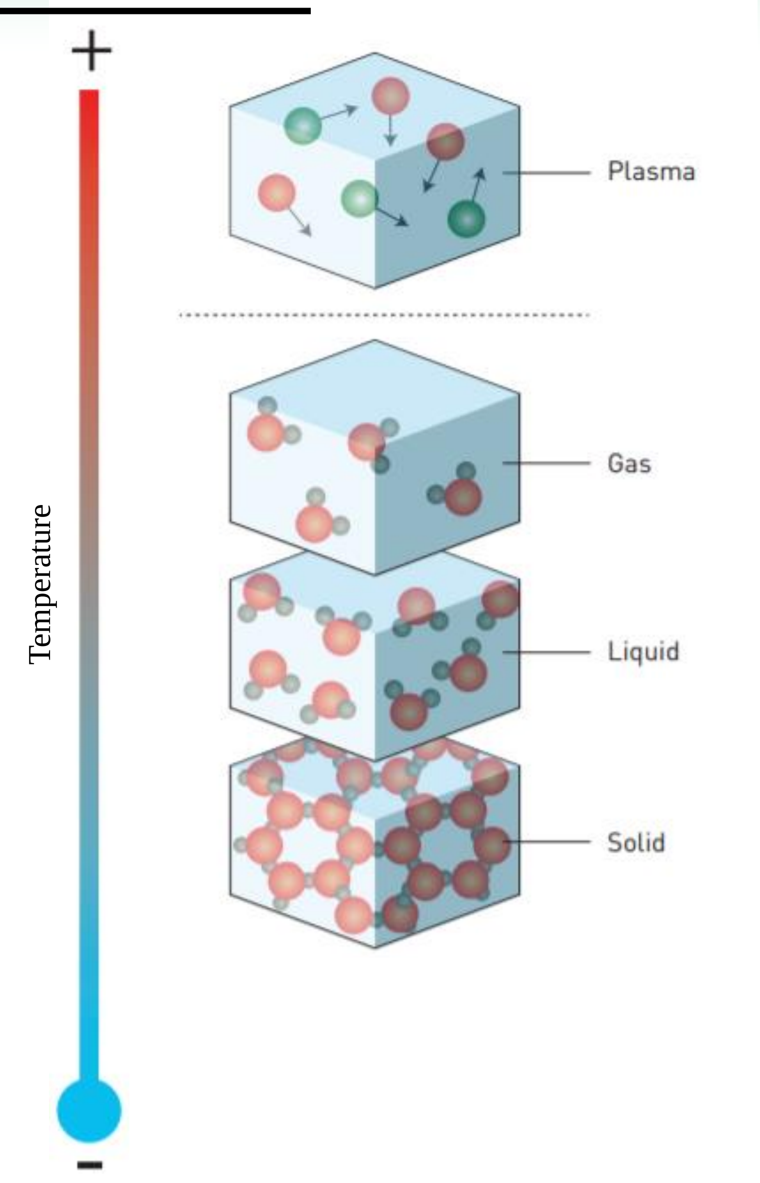
(n=3)

Google images: Bagel
Google images: PhD mug
Google images: Pretzel
Google images: Spinner

Classical phases and Quantum phases

- Classically, all matter exists as either a solid, a liquid, a gas, or a plasma.
  - Phase transitions occurs when matter changes from one form to another.
  - Quantum effects are washed out by thermal fluctuations.
- Near absolute zero, matter takes on strange new phases and quantum effects become very pronounced.
  - Superconductor – zero electrical resistance when cooled below a critical temperature
  - Superfluid – zero viscosity (resistance of a fluid to flow) below a critical temperature. E.g. ^{4}He



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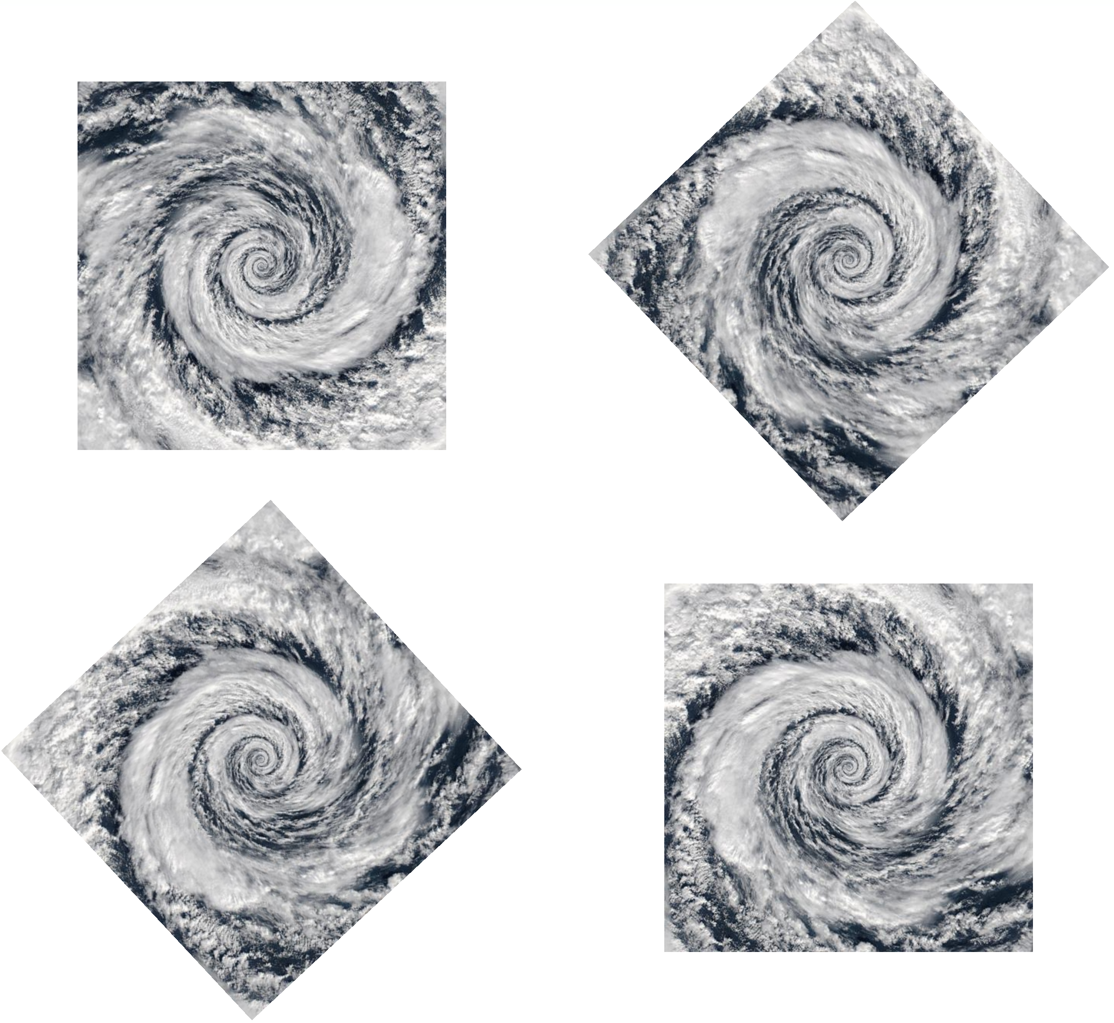
Thin films

- It was assumed that phase transitions could not occur in thin films (e.g. thin liquid layer of helium) even at absolute zero due to thermal noise.

- In 1972 Kosterlitz and Thouless identified a completely new type of phase transition in such extremely thin layers, where topological defects play a crucial role.

  - Occurs due to the configurations of tiny vortices of electronic spins on 2D surfaces.

    - Vortices are topological.



https://giphy.com/gifs/clouds-hurricane-fractal-d3mlXPjoK1ROfr9u

What is the Kosterlitz-Thouless transition?

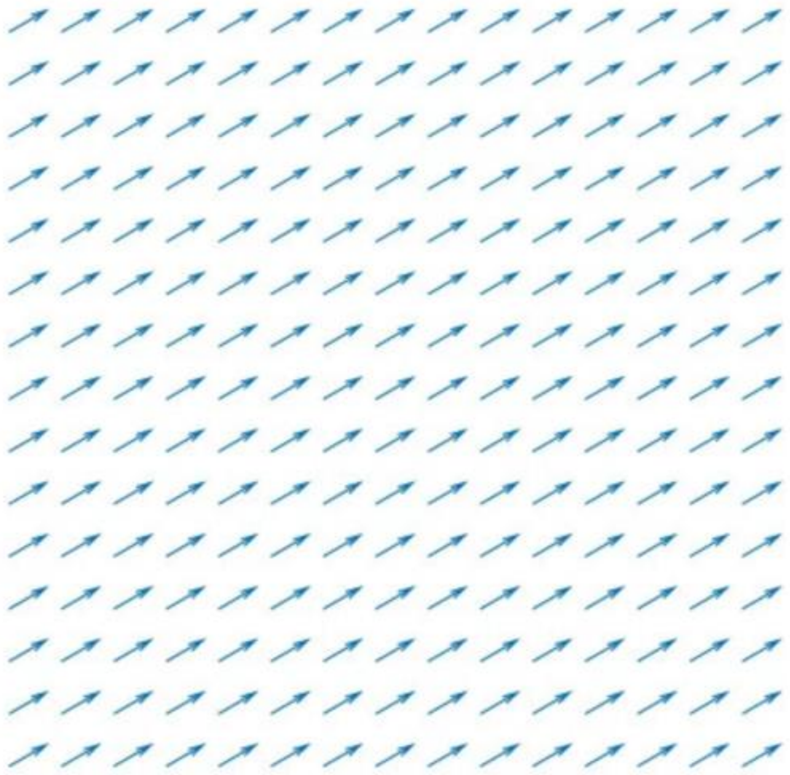
- Classical XY model in 2D.

- At zero temperature, minimize energy when all the arrows point in the same direction. (Ferromagnetic states.)
  - Infinite number of such states
    - Assume all the spins point along the X direction, then by turning all the spins some angle, the system's energy changes slightly
    - Implies: create excitations with small energy (Goldstone modes – if system was ordered).

- Spin wave analysis:
  - At finite but small temperatures, low-energy excitations destroy the order (spin alignment) – agreement with Mermin-Wagner theorem

  - But, the system has long-range correlations (that decay algebraically) due to the low-energy excitations.

  - Not valid at high temperatures (totally disorded system with short-range correlations).



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What is the Kosterlitz-Thouless transition?

- What's missing in the spin-wave analysis is the possibility of **vortices** (closed loop on lattice, spin angles add to multiples of 2π).

- By heating, more energy is gained.

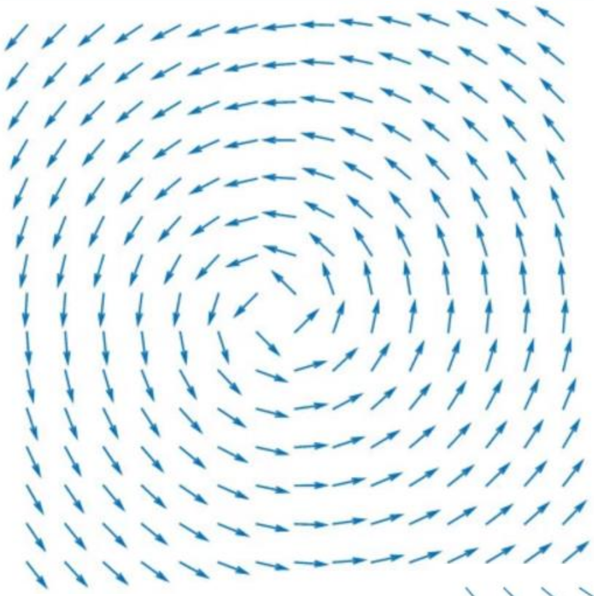
- Spins can now change direction.

  - One possibility is a vortex.
    - High energy spin excitations
      - Vortices considered irrelevant.

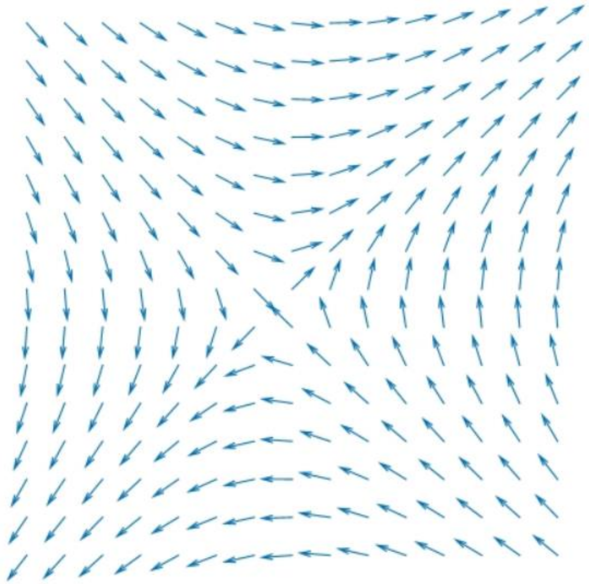
  - Another possibility, is an 'antivortex':
    - High energy spin excitations
      - Antivortices considered irrelevant

- Important for understanding the transitions from long-range correlations at low temperatures to short-range correlations at high temperatures.

Vortex:



Antivortex:



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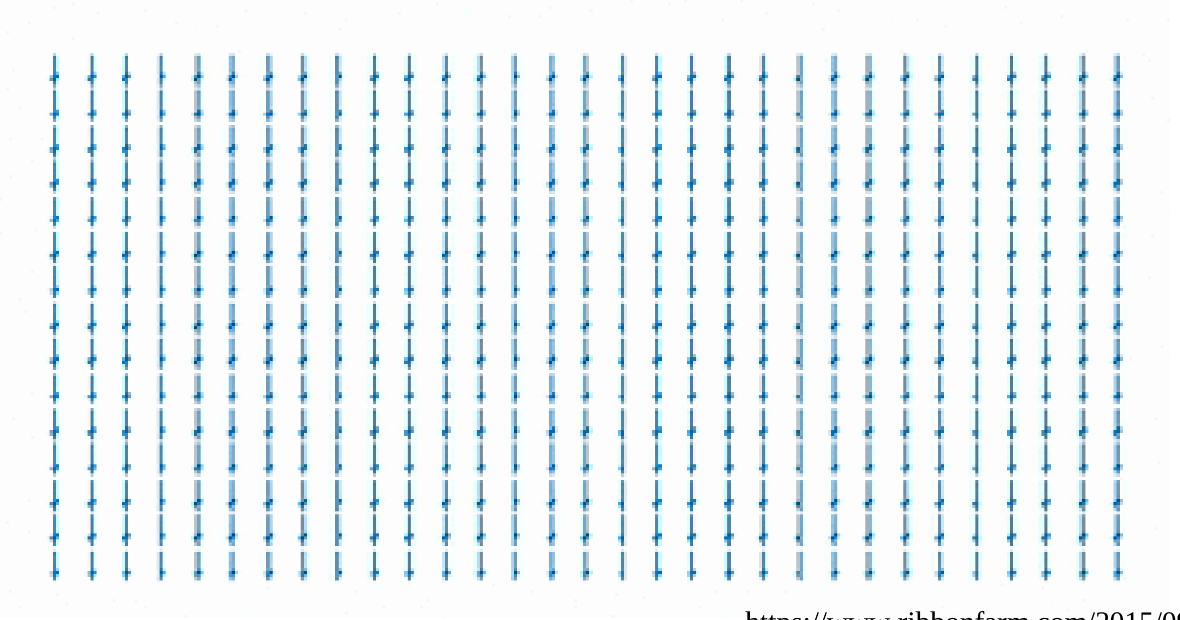
What is the Kosterlitz-Thouless transition?

- Here’s what Kosterlitz and Thouless noticed:

  - Combination of *a vortex together with an antivortex* has much less energy than either one alone.

  - When hot enough, the spins will form ‘vortex-antivortex pairs’.

  - Vortex-antivortex pairs move around and bump into each other.
    - They can also appear or disappear.
      - Topological excitation because cannot undo a vortex (antivortex) by locally changing the orientation of spins



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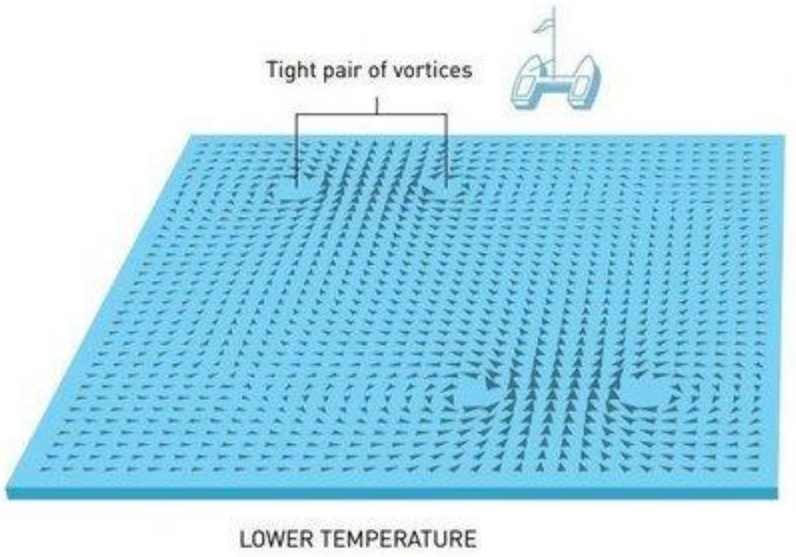
What is the Kosterlitz-Thouless transition?

- At low temperatures, few vortex-antivortex pairs (little energy).

  - Spin vortices are tightly bonded.
    - Requires a lot of energy to separate them.
    - Bounded pairs do not affect correlations at long distances



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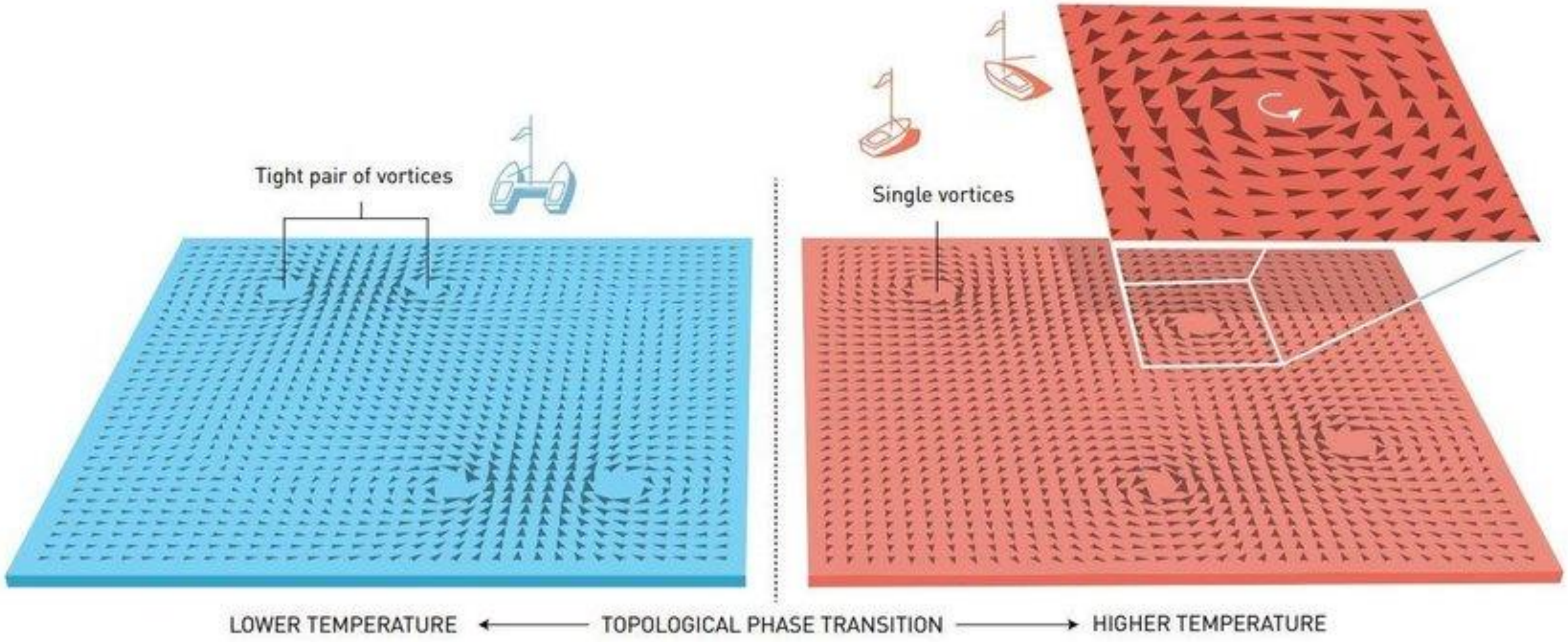
What is the Kosterlitz-Thouless transition?

- When heated, more vortex-antivortex pairs appear (more energy available).
  - Distance between the vortices and antivortices grows further apart.
  - Kosterlitz and Thouless showed that there’s a certain temperature at which the vortex-antivortex pairs suddenly ‘unbind’ and break apart.
    - All the vortices and antivortices are free to move (destroys the correlations between distant spins)
    - This triggers a quantum phase transition from one state of matter to another.



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Applications

- Kosterlitz and Thouless went on to
  - Calculate that the phase transition would occur at relatively high temperatures, above which superconductivity would disappear.
  - Show that superconductivity or superfluidity can occur in 2D layers at low temperatures.
- Kosterlitz-Thouless transitions
  - Led to strange new states of matter, called ‘topological phases’.
  - Used to study superconductors and superfluids.
  - Applied to phase transitions that occur when a ferromagnetic thin film is cooled below the Curie temperature and the spins line up, giving rise to net magnetization.

Conclusion

- The Kosterlitz–Thouless transition is the sudden unbinding of the vortex-antivortex pairs as a thin film is heated.
    - This is a completely new phase transition driven by topological effects.