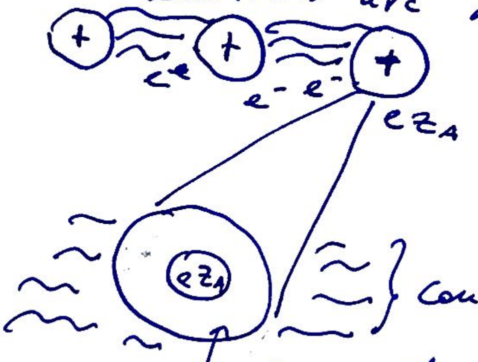
Drude theory of metals

Basics of Drude theory of metals.

- The theory of metalic conductivity – 1900.
  electric + thermal

- Drude knew about electrons as negative particles inside solids from J. J. Thomson.
  to compensate the negative charge there should be positive particles in a metal. and immobile

e^- electrons are detached



$e = 4.8 \cdot 10^{-10}$ esu $= 1.6 \cdot 10^{-19}$ C

Z conduction electrons

~~nucleus~~ core electrons $- Z + Z_A$

$eZ_A - eZ = e(Z_A - Z)$

core electrons

Drude treated the conduction electrons as GAS of non-interacting "atoms" on the electrostatic background of an ionic core.

Fast forward to more modern times:

there are $6.02 \cdot 10^{23}$ atoms/mole $= N_A$

if we know density $\rho = [g/cm^3] \rightarrow$

$\rho/A = [moles/cm^3]$ if each atom contributes Z electrons: Total # of electrons/cm^3

(A ← atomic weight)

$$\boxed{\frac{N}{V} = N_A \cdot Z \frac{\rho}{A} = \text{carrier density} = n}$$

e.g. $n_{Li} = 6 \cdot 10^{23} \dfrac{Z=1 \cdot 0.5\, g/cm^3}{A=7} = \dfrac{6 \cdot 5 \cdot 10^{-1} \cdot 1 \cdot N_A}{7} \approx 5 \cdot 10^{22} \dfrac{e}{cm^3}$

$n_{Be} = 24.7$, $n_{Fe} = 17.0$ $n_{Cs} = 0.91$

Drude introduced electron density:

r_s: (circle with radius r_s) = Volume per cond. electron $\frac{1}{n}$

$r_s : \frac{1}{n} = \frac{V}{N} = \frac{4\pi}{3} r_s^3 \Rightarrow \boxed{r_s = \sqrt[3]{\frac{3}{4\pi} \cdot \frac{1}{n}}}$

$r_s \sim 10^{-8}\, cm = 10^{-8} \cdot 10^{-2}\, m = 10^{-10}\, m = 1\, \mathring{A}$

we can scale r_s to the radius of H atom:

$a_0 = \frac{\hbar^2}{me^2} = 0.529 \cdot 10^{-8}\, cm = 0.52 \cdot 10^{-10}\, m = 0.52\, \mathring{A}$

e.g. in many cases $r_s/a_0 \sim 2 \div 3$

Despite e–e and e–ion interactions they are ignored.

①

no interactions between collisions.

② no external B and E electron moves straight

{ if external field then electron moves according to Newton's Law without interactions with other e^-

known today as

$\boxed{\text{the independent electron approximation}}$

③ The role of collisions is to instantly change the velocity and direction of an electron

Drude thought of bouncing off the ionic cores.

Suprizingly e-e interaction in normal metals is NOT important.

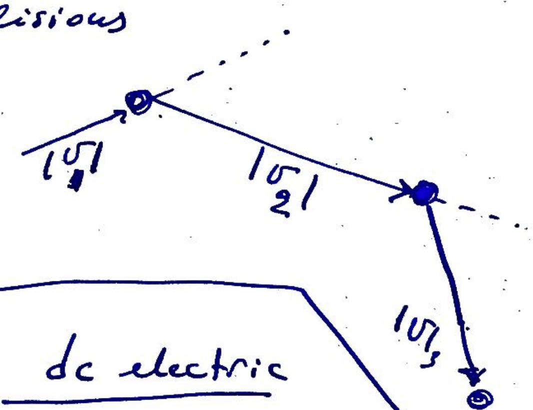
* ④ Collissions happen with the probability of $1/\tau$ (in units of time)

More physicaly it means an electron will travel on average τ units of time between the collisions

$\tau \equiv$ the relaxation time $\equiv$ the collision time $=$

$\equiv$ the mean free time

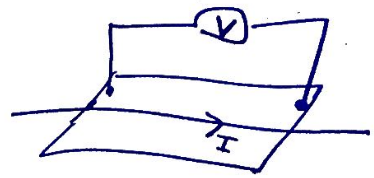
⑤ Electrons lose their energy via collisions



velocitice and directions before and afthe the collisions are uncelated

the speed ~ temperature at the position of a collision. e.g. in hot regions collisions are faster and τ is smaller.

Theory of dc electric conductivity



According to the Ohm's law

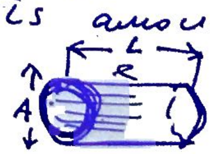
$V = IR$

Resistance R is a function of wire size

Can we explain this?

To avoid issues with the wire size lets introduce ρ = resistivity such as

$\bar{E} = \rho \, \bar{j}$ $\quad$ j is amount of charge per second per crossing area



so $j = \frac{I}{A}$ and $V = E \cdot L$
potential drop across wire

from $E = \rho j$, $J = \frac{I}{A}$ $E = \frac{V}{L}$ $\Big| \Rightarrow \frac{V}{L} = \rho \frac{I}{A} \Rightarrow$

$$\frac{V}{I} = \boxed{\rho \frac{L}{A} = R}$$

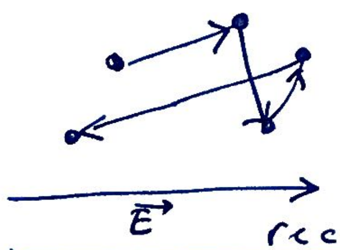
More microscopically, if n electrons move with $\bar{v}$: $v \cdot dt$ is the distance they cover in dt

so $n \cdot \underbrace{(v dt) A}_{dV}$ e^- will cross the volume ΔV, the total charge crossing this volume : $- e n (dt \cdot v) \cdot A$

The current density then $\boxed{\bar{j} = - e n \bar{v}}$ $[e^- / \text{sec} \cdot cm^2]$

v - is the average velocity which is $\simeq 0$, no net electric charge. If $\bar{E} \neq 0 \Rightarrow \bar{j}$ is not 0 b/c $\bar{v} \neq 0$!

Lets move to even more microscopic view.



t is the time since last collision

v_0 - after the collision

$v_0 - \frac{eE}{m} t$ $\Rightarrow$ lets average this over many collissions

recall

$$\boxed{F = ma = m \frac{v}{t} = qE}$$

$$v_{avr} = \underbrace{\langle v_0 \rangle}_{=0} - \langle \frac{eE}{m} t \rangle \quad t \to \tau$$

$$v_{avr} = - \frac{eE}{m} \tau$$

and since $\bar{j} = - e n \bar{v} = - e n \left(- \frac{e \bar{E}}{m} \tau \right) = \left(\frac{n e^2 \tau}{m} \right) \bar{E}$

recall $\bar{E} = \rho \bar{j}$ $\Rightarrow$ $\bar{j} = \frac{1}{\rho} \bar{E}$ $\Rightarrow \boxed{\bar{j} = \sigma \bar{E}}$

thus sigma = conductivity = $\boxed{\frac{n e^2 \tau}{m} = \sigma}$

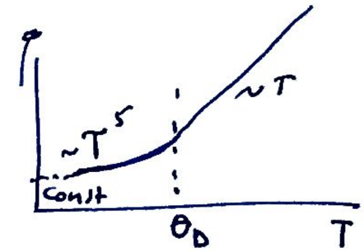
Now: we know n from the simple estimates $n = N_A \frac{Z \rho_m}{A}$

$\frac{e^2}{m}$ = are constants , what is not known is τ.

That is why often we use transport measurents to quantify $\tau = \frac{\sigma m}{n e^2}$

Typical numbers

	ρ [$\mu\Omega$ cm]		ρ(RT/77K)
Li	RT	77K	
	12.4	1.04	~ 12
Au	2.84	0.5	~ 5.6
Bi	156	35	~ 5



We can express ~~the~~ ρ in terms of τ by using r_s

$$\tau = \left(\frac{0.22}{\rho_\mu}\right)\left(\frac{r_s}{a_0}\right)^3 10^{-14}\ \text{sec} \qquad \rho_\mu\ [\mu\Omega\,\text{cm}]$$

From this we can conclude that for typical materials $\tau \sim 10^{-14} - 10^{-15} = 10 \div 1$ fsec at RT.
Mean free path then can be expressed as:

$$l_{mfp} = v_0 \cdot \tau =$$

$$\downarrow$$

$$\frac{m v_0^2}{2} = \frac{3}{2} k_B T \Rightarrow v_0 = \sqrt{\frac{3k_B}{m} \cdot T} \sim 10^7 \frac{\text{cm}}{\text{sec}} \text{ at RT}$$

$$l_{mfp} \sim 10^7 \cdot 10^{-14} = 10^{-7} \div 10^{-8}\ \text{cm} \cong 1\,\text{Å}$$

So since $1\,\text{A} \sim a$ = lattice size may be it's true that e^- collides with the positive ions.
Since we don't have a theory of τ (by the way experimentally it's measured very precisely by angular resolved photoemission ARPES) we focus on quantities which are τ independent!

Case 1: Metals and dielectrics in d.c. field
Case 2: —— " —— in a.c. field.

Consider case 1: $\bar{v} = \bar{P}(t)/m \Rightarrow \bar{j} = -en\bar{v} = -en\,\bar{P}(t)/m$

Since we know p at t, what is p at $t+dt$

Now { electron scatters within time dt with probability $\frac{dt}{\tau}$

and not scattering $1-\frac{dt}{\tau}$

e^-

Scatter $\frac{dt}{\tau}$ ↓

Contribution of those not scattered e^- is

$P(t) + f(t)\,dt$

↑ average mom. ↑ extra kick due to f

not scatter $1-\frac{dt}{\tau}$ ↓

moves under application of the external force $\bar{f}(t)$ and acquire an extra momentum $\bar{f}(t)\cdot dt$

$\Downarrow$

$$\bar{P}(t+dt) \cong \left(1-\frac{dt}{\tau}\right)\left[\bar{p}(t) + \bar{f}(t)\right] = P(t) - \frac{dt}{\tau}p(t) + f(t)\,dt$$

the correction to this eqn. from scattered carries is $O(dt)^2$, i.e. scattered electrons contribute only this fraction $\frac{dt}{\tau}$ since the collission is random the e^- only remembers the last event: $f(t)\,dt$, thus we have $\frac{dt}{\tau}\cdot f(t)\,dt \Rightarrow O(dt)^2$

we ~~ignore~~ ignore the scattered electrons

$$p(t+dt) - p(t) = -\frac{dt}{\tau}p(t) + f(t)\,dt$$

$$\frac{p(t+dt)-p(t)}{dt} = \boxed{\frac{d\bar{p}}{dt} = -\frac{\bar{p}(t)}{\tau} + \bar{f}(t)}$$

the effect of the external force is to dump the momentum $\bar{P}(t)$ by τ.

where $\omega_c = \frac{eH}{mc}$; $0 = -eE_x - \omega_c p_y - p_x/\tau$

$0 = -eE_y + \omega_c p_x - p_y/\tau$

Recall $\bar{j} = -ne\bar{v}$ $\quad \bar{j}\cdot m = -ne\,m\bar{v} = -ne\bar{p} \Rightarrow p = \frac{\bar{j}\,m}{en}$ -

so $\left.\begin{array}{l} p_x = -\frac{j_x\,m}{en} \\ p_y = -\frac{j_y\,m}{en} \end{array}\right| \Rightarrow \begin{cases} 0 = -eE_x + \frac{\omega_c \cdot m}{en}\cdot j_y + \frac{j_x\,m}{en\tau} \\ 0 = -eE_y - \omega_c \frac{j_x m}{en} + \frac{j_y m}{en\tau} \end{cases}$

also recall: $\sigma = \frac{ne^2\tau}{m}$

$$\begin{cases} 0 = -\underbrace{\frac{e^2 n\tau}{m}}_{\sigma_0} E_x + \omega_c\tau j_y + j_x \\ 0 = -\underbrace{\frac{e^2 n\tau}{m}}_{\sigma_0} E_y - \omega_c\tau j_x + j_y \end{cases}$$

$$\begin{cases} \sigma_0 E_x = \omega_c\tau j_y + j_x \\ \sigma_0 E_y = -\omega_c\tau j_x + j_y \end{cases}$$

In the steady state $j_y = 0\,! \Rightarrow$

$$\begin{cases} \sigma_0 E_x = j_x & \leftarrow \text{kind of trivial} \\ \sigma_0 E_y = -\omega_c\tau j_x & \Rightarrow E_y = -\frac{\omega_c\tau}{\sigma_0} j_x \end{cases}$$

and $\left.\begin{array}{l} \text{from } \omega_c = \frac{eH}{mc} \\ \text{and } \sigma = -\frac{e^2 n\tau}{m} \end{array}\right| \Rightarrow E_y = -\left(\frac{H}{nec}\right) j_x \Rightarrow$

$R_H = \frac{E_y}{j_x H} \Rightarrow R_H = -\frac{\cancel{H}}{nec}\cdot\frac{\cancel{j_x}}{\cancel{j_x}}\cdot\frac{1}{\cancel{H}} = -\frac{1}{nec}$

$$\boxed{R_H = -\frac{1}{nec}}$$

~~Strangely~~ Oddly enough, R_H depends on H, T etc.

Why? it turned out the value of R_H where n is calculated from the valence electron participating in conduction is only valid for ultra pure elemental metals at very low T and high magnetic field H.

Another comment: $\omega_c \tau$ can be a good measure of the external magnetic field.

i.e. when $\omega_c \tau$ is small, e.g. $\sigma_0 E_x = \underbrace{\omega_c \tau}_{\text{small} \to 0} j_y + j_x$

$$\sigma_0 E_y = -\underbrace{\omega_c \tau}_{\to 0} j_x + j_y$$

So $\bar{j}$ and $\bar{E}$ are almost parallel to each other. However, when ω_c increases the angle between E_x and E_y gets larger.

$$\text{HALL ANGLE} \equiv \tan\theta = E_y / E_x = \frac{j_x H}{nec E_x} = \frac{\sigma E_x H}{nec E_x} = \frac{\cancel{n} e^2 \tau H}{\cancel{n} e c m} = -\omega_c \tau$$

i.e. long relaxation time implies large Hall angle θ.

For experiment: a good estimate of ν_c:

$$\nu_c(\text{GHz}) = 2.8 \cdot H(\text{kG}), \quad \nu_c = \frac{\omega_c}{2\pi}$$

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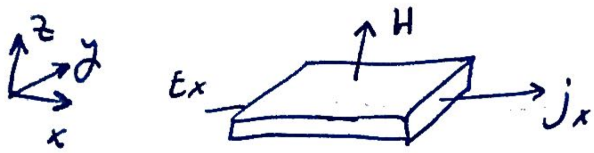
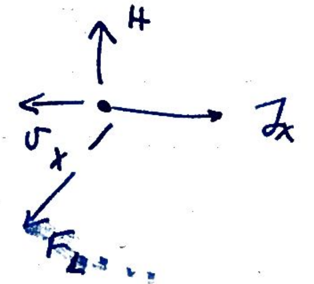
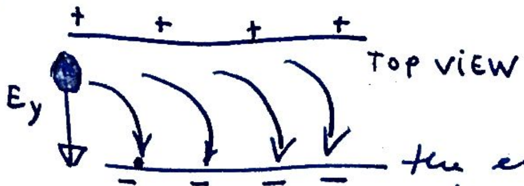
Hall effect.

A non-magnetic material placed into a magnetic field, when we ~~[illegible]~~ run current accross it:





$\bar{F}_L = -\frac{e}{c}\bar{V}\times B$



the edge of the sample or surface

In the equalibrium $F_L = F_{HALL}$, due to the surface build up

To quantify this phenomenon Hall introduced 2 ratios:

$$\frac{E_x}{j_x} = \rho(H) \equiv \text{magnetoresistance}$$

$$\text{Hall coeff.} = \boxed{R_H = \frac{E_y}{j_x H}}$$

since E_y is negative R_H is negative too.

RULE: For electrons $R_H < 0$
holes $R_H > 0$

by measuring the sign of the Hall coef. ~~[illegible]~~ one can tell what kind of carriers are dominant at the Fermi edge.

To calculate $\rho(H)$ and R_H we can go back to

$$\frac{dp}{dt} = \underbrace{-e\left(E + \frac{p}{mc}\times H\right)}_{f} - p/\tau$$

$$\frac{dp_x}{dt} = -(e E_x + \omega_c p_y) - p_x/\tau$$

$$\frac{dp_y}{dt} = -e(E_y + \omega_c p_x) - p_y/\tau$$

in the steady condition $\frac{d\bar{p}}{dt} = 0$

END