Solutions to the homework #7 (Due March 24)

① Griffiths, 4.32

(a) $\Sigma^{*0} \to \Sigma^+ + \pi^-$

(b) $\Sigma^{*0} \to \Sigma^0 + \pi^0$

(c) $\Sigma^{*0} \to \Sigma^- + \pi^+$

Due to $\Sigma^{*0} = |1;0\rangle$, $\pi^- = |1;-1\rangle$, we have

$\Sigma^+ = |1;1\rangle$, $\pi^0 = |1;0\rangle$

$\Sigma^0 = |1;0\rangle$, $\pi^+ = |1;1\rangle$

$\Sigma^- = |1;-1\rangle$

$M = \langle \psi_f | \hat{H} | \psi_i \rangle$

(a) $\psi_f = |1,1\rangle |1;-1\rangle = \sqrt{\frac{1}{6}}\,|2;0\rangle + \sqrt{\frac{1}{2}}\,|1;0\rangle + \sqrt{\frac{1}{3}}\,|0;0\rangle$

(b) $\psi_f = |1;0\rangle\,|1;0\rangle = \sqrt{\frac{2}{3}}\,|2;0\rangle + 0\,|1;0\rangle - \sqrt{1/3}\,|0;0\rangle$

(c) $\psi_f = |1;-1\rangle\,|1;1\rangle = \sqrt{1/6}\,|2;0\rangle - \sqrt{1/2}\,|1;0\rangle + \sqrt{1/3}\,|0;0\rangle$

$\psi_i = |1;0\rangle$

Hence $M_a = 1/2 = M_c$, but $M_b = 0$.

Finally from 100 disintegrations 50 will be in (a) decay, and 50 in (c)
There are no disintegrations in (b).

② Griffiths, 4.37

$\eta \to 2\gamma$; $\eta \to 3\pi$; $\eta = \pi\pi\gamma$

(a) $\eta \to 2\pi$ is forbidden for strong and e/m interactions, because of "P-parity": for η: $P_\eta = -1$
for 2π: $P_{2\pi} = (-1)^n = 1$ $\Big\}\ P_i \neq P_f$

and because of Charge conjugation x Parity (CP) violation:

$CP|\eta\rangle = (+1)(-1) = -1$

$CP|2\pi\rangle = 1.$

(b) $\eta \to 3\pi$ is forbidden as strong interaction and allowed as e/m, because "P-parity" is conserved $P_\eta = -1 = P_{3\pi} = (-1)^n = (-1)^3 = -1$, and C-charge conjugation is conserved, but strangeness of the reaction is not conserved.

3.*Griffiths 5.18*

The experimental results say that

$\Gamma(\psi \to hadrons) : \Gamma(\psi \to e^+e^-) : \Gamma(\psi \to \mu^+\mu^-) = 88\% : 6\% : 6\%$

Due to $\Gamma = \sigma\upsilon|\psi(0)|$, let's compare $\sigma(\psi \to hadrons) : \sigma(\psi \to e^+e^-) : \sigma(\psi \to \mu^+\mu^-)$.

$$\sigma(\psi \to hadrons) = \frac{16}{9}\left(\pi^2 - 9\right)\frac{5}{18}\alpha_S{}^3\frac{\hbar^2}{m_c{}^2c^2}\frac{c}{\upsilon}$$

$$\sigma(\psi \to e^+e^-) = \sigma(\psi \to \mu^+\mu^-) = \frac{16\pi}{3}\left(\frac{q_c q_e}{m_c c^2}\right)^2 \frac{c}{\upsilon}$$

Finally we have: $\Gamma(\psi \to hadrons) : \Gamma(\psi \to e^+e^-) : \Gamma(\psi \to \mu^+\mu^-) =$

$$\frac{16}{9}\left(\pi^2 - 9\right)\frac{5}{18}\alpha_S{}^3 \frac{\hbar^2}{m_c{}^2 c^2} : \frac{16\pi}{3}\left(\frac{q_c q_e}{m_c c^2}\right)^2 : \frac{16\pi}{3}\left(\frac{q_c q_e}{m_c c^2}\right)^2 =$$

$$\sim 0.424\alpha_S{}^3\hbar^2 : 16.747\frac{(q_c q_e)^2}{c^2} : 16.747\frac{(q_c q_e)^2}{c^2} = 0.424\alpha_S{}^3\hbar^2 : \frac{16.747}{c^2}\left(\frac{2}{3}\times 1e^2\right)^2 : \frac{16.747}{c^2}\left(\frac{2}{3}\times 1e^2\right)^2 =$$

$$0.424\alpha_S{}^3 : \left(\frac{e^2}{\hbar c}\right)^2 7.411 : \left(\frac{e^2}{\hbar c}\right)^2 7.411 = 4.24\times 10^{-4} : 7.411\alpha^2 : 7.411\alpha^2 = 4.24\times 10^{-4} : 3.97\times 10^{-4} : 3.97\times 10^{-4} =$$

$1.07 : 1 : 1 = 35\% : 32.5\% : 32.5\%$, Which is not consistent with the experiment.

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(b) For "beautiful" mesons:

$$M(b\bar{u}) = m_b + m_u - 3\frac{m_u^2}{m_u m_b}\times 160 = 4700 + 310 - 480\frac{310}{4700\cdot 310} \approx \boxed{4978\ MeV/c^2}$$

The experiment result: $\boxed{M(b\bar{u}) = 5271\ MeV/c^2}$

$$M((b\bar{u})^*) = m_b + m_u + \frac{1}{4}\times 160\frac{(2\cdot m_u)^2}{m_b m_u} = \boxed{5021\ MeV/c^2}$$

$$M(b\bar{d}) = M(b\bar{u}) = \boxed{4978\ MeV/c^2}$$

$$M((b\bar{d})^*) = M((b\bar{u})^*) = \boxed{5021\ MeV/c^2}$$

$$M(b\bar{s}) = m_b + m_s - \frac{3\times 160\times m_u^2}{m_b m_s} = \boxed{5163\ \frac{MeV}{c^2}}$$

$$M((b\bar{s})^*) = m_b + m_s + \frac{160\times m_u^2}{m_b m_s} = \boxed{5189\ MeV/c^2}$$

$$M(b\bar{c}) = m_b + m_c - 3\times m_u^2\times 160/(m_b m_c) = \boxed{6193\ MeV/c^2}$$

$$M((bc)^*) = m_b + m_c + m_u^2\times 160/(m_b m_c) = \boxed{6202\ MeV/c^2}$$

$$M(b\bar{b}) = 2m_b - 3m_u^2\times 160/m_b^2 = \boxed{9398\ MeV/c^2}$$

$$M((b\bar{b})^*) = 2m_b + m_u^2\times 160/m_b^2 = \boxed{9401\ MeV/c^2}$$

④ Griffiths 5.23

$$\mathcal{M}(\text{meson}) = m_1 + m_2 + A \cdot \frac{(\vec{S}_1 \cdot \vec{S}_2)}{m_1 m_2}$$, for pseudoscalars $\vec{S}_1 \cdot \vec{S}_2 = -\frac{3}{4}\hbar^2$
for scalars $\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{4}\hbar^2$

(a) $$\mathcal{M}(\eta_c(c\bar{c})) = 2m_c + \frac{\vec{S}_c \cdot \vec{S}_{\bar{c}}}{m_c^2} \cdot \left(\frac{2m_u}{\hbar}\right)^2 160\ \frac{MeV}{c^2} = 2 \cdot 1.5 \times 10^3 MeV + \frac{\left(-\frac{3}{4}\right) \cdot 4 \cdot 160 MeV}{(1500\ MeV)^2}$$

$$= 2 \times (1.5 \times 10^3 MeV) - \frac{3}{4} \times 160 MeV\ \frac{(2 \times 310 MeV)^2}{(1500 MeV)^2} \approx \boxed{2979.5 MeV}$$

From an experiment $\boxed{M_{\eta_c} = 2979.8 \pm 2.1\ MeV}$ $\Rightarrow$ Good agreement with experiment!

where $c=1$

$$\mathcal{M}(D(c\bar{u})) = m_c + m_u - \frac{3}{4} \times 160 MeV \left[\frac{610^2}{m_u \cdot m_c}\right] = 1500 + 310 - \frac{3}{4} \times 160 \frac{610^2}{1500 \cdot 310} \approx \boxed{1711 \frac{MeV}{c^2}}$$

The experimental result: $\boxed{M_D = 1865\ MeV/c^2}$

$$\mathcal{M}(F(c\bar{s})) = m_c + m_s - 3 \times 160 \times \frac{310^2}{m_c m_s} = 1500 + 483 - 3 \times 160 \times \frac{310^2}{1500 \cdot 483} \approx \boxed{1920\ MeV/c^2}$$

The experiment result: $\boxed{M_F = 1971\ MeV/c^2}$

$$\mathcal{M}(\psi(c\bar{c})) = 2m_c + \frac{1}{4} \times 160 \left(\frac{2 \cdot 310}{m_c}\right)^2 = 2 \times 1500 + 40\left(\frac{620}{1500}\right)^2 = \boxed{3007\ MeV/c^2}$$

From experiments: $\boxed{M_\psi \simeq 3097\ MeV/c^2}$

$$\mathcal{M}(D^*(c\bar{u})) = m_c + m_u + 160 \left(\frac{310^2}{m_c m_u}\right) = \boxed{1843\ MeV/c^2}$$ Experiment $\boxed{M_{D^*} = 2010 \frac{MeV}{c^2}}$

$$\mathcal{M}(F^*(c\bar{s})) = m_c + m_s + 160 \times \frac{310^2}{m_c m_s} = \boxed{2004\ MeV/c^2}$$

⑤ Griffiths, 5.27

The spin-flavor wave function for $\overset{uus}{\Sigma^+}|\uparrow\rangle$ and $\overset{uds}{\Lambda}|\downarrow\rangle$: mixed symmetry $J = 1/2$

$$|\Sigma^+ : \tfrac{1}{2}; \tfrac{1}{2}\rangle = \psi_{\Sigma^+}(\text{spin})\, \psi_{\Sigma^+}(\text{flavor}) =$$

$$= \frac{\sqrt{2}}{3}\Big(\psi_{12}(\text{spin})\psi_{12}(\text{flavor}) + \psi_{23}(\text{spin})\psi_{23}(\text{flavor}) + \psi_{31}(\text{spin})\psi_{31}(\text{fl})\Big) =$$

$$= \frac{\sqrt{2}}{3}\Big[\tfrac{1}{2}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(usu - suu) + \tfrac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uus - usu) + \tfrac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uus - suu)\Big] =$$

$$= \frac{1}{3\sqrt{2}}\Big[2u(\uparrow)u(\uparrow)s(\downarrow) - u(\uparrow)u(\downarrow)s(\uparrow) - u(\downarrow)u(\uparrow)d(\uparrow)\Big] + \text{permutations.}$$

$$|\Lambda : \tfrac{1}{2}; -\tfrac{1}{2}\rangle = \frac{1}{2\sqrt{2}}\ \frac{\sqrt{3}}{2}\Big[(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)(usd - sud + dsu - sdu) +$$
$$+ (\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow)(uds - sdu + uds - usd) + (\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow)(uds - dsu + uds - usd)\Big]$$

⑥ Griffiths, 5.32

The mass of the baryon can be found from the equation:

$$M = m_1 + m_2 + m_3 + A' \left[\frac{\vec{S}_1 \vec{S}_2}{m_1 m_2} + \frac{\vec{S}_1 \cdot \vec{S}_3}{m_1 m_3} + \frac{\vec{S}_2 \cdot \vec{S}_3}{m_2 m_3} \right]$$

Ξ baryon has a constuction uss: $I = 1/2$, $S = -2$, hence

$$M_{\Xi} = 2m_s + m_u + \frac{\hbar^2}{4} \left(\frac{2m_u}{\hbar} \right)^2 \times 50 \frac{MeV}{c^2} \left[\frac{1}{m_s^2} - \frac{4}{m_u m_s} \right] =$$

$$= 2 \times 538 + 363 + 363^2 \times 50 \left[\frac{1}{538} - \frac{4}{538 \cdot 363} \right] = \boxed{1327 \frac{MeV}{c^2}}$$

Experiment: $M_{\Xi} = 1318 \frac{MeV}{c^2}$.