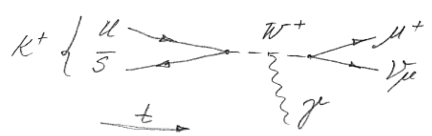
Solutions to the homework #5 (due Wednesday, March 3)

(1) Griffiths. 2.7

(a) $p+\bar{p}\to\pi^{+}+\pi^{0}$: impossible. Violation of charge conservation.
(b) $\eta\to\gamma+\gamma$: possible. Electromagnetic interaction.
(c) $\Sigma^{0}\to\Lambda+\pi^{0}$: impossible. Violation of energy conservation.
(d) $\Sigma^{-}\to n+\pi^{-}$: possible. Weak interaction.
(e) $e^{+}+e^{-}\to\mu^{+}+\mu^{-}$: possible. Electromagnetic interaction.
(f) $\mu^{-}\to e^{-}+\bar{\nu}_{e}$: impossible. Lepton number is not conserved.
(g) $\Delta^{+}\to p+\pi^{0}$: impossible. The total angular momentum J is not con.
(h) $\bar{\nu}_{e}+p\to n+e^{+}$: possible. Weak interaction.
(i) $e+p\to\nu_{e}+\pi^{0}$: impossible. Baryon number is not conserved.
(j) $p+p\to\Sigma^{+}+n+K^{0}+\pi^{+}+\pi^{0}$: Possible. Weak interaction.
(k) $p\to e^{+}+\gamma$: impossible. Violation of lepton number.
(l) $p+p\to p+p+p+\bar{p}$: possible. Strong interaction.
(m) $n+\bar{n}\to\pi^{+}+\pi^{-}+\pi^{0}$: possible. Strong interaction.
(n) $\pi^{+}+n\to\pi^{-}+p$: impossible. Violation of charge conservation.
(o) $K^{-}\to\pi^{-}+\pi^{0}$: possible. Weak interaction.
(p) $\Sigma^{+}+n\to\Sigma^{-}+p$: impossible. Violation of charge conservation.
(q) $\Sigma^{0}\to\Lambda+\gamma$: possible. Strong interaction.
(r) $\Xi^{-}\to\Lambda+\pi^{-}$: possible. Strong interaction.
(s) $\Xi^{0}\to p+\pi^{-}$: impossible, $\Delta S=\Delta Q=2$, forbidden S_2 mode.
(t) $\pi^{-}+p\to\Lambda+K^{0}$: possible. Strong interaction.
(u) $\pi^{0}\to\gamma+\gamma$: possible. Strong interaction.
(v) $\Sigma^{-}\to n+e+\bar{\nu}_{e}$: possible. Weak interaction.

(2) Griffiths. 2.8

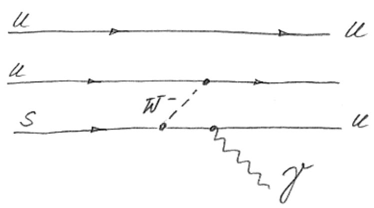
(a) $K^{+}\to\mu^{+}+\nu_{\mu}+\gamma$. The Feynman diagram for the process is:



There are weak and electromagnetic forces.

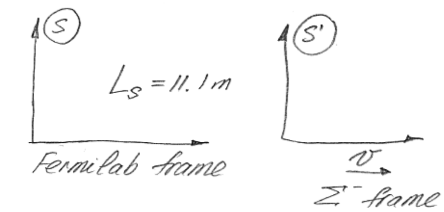
(γ line can be at any point, there are 5 possibilities for it)

(b) $\Sigma^{+}\to p+\gamma$ The Feynman diagram is



Again, γ can be at any point. There are 6 possibilities for γ.

(3) According to Lorentz' relativity equations $t_{lab} = \gamma t_{proper}$, where t_{proper} is a time measured in the particle frame, hence lifetime measured by the observer $T_{lab} = \gamma T_{part.}$ But $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$, hence $\boxed{T_{lab} > T_{part.}}$

4) 

Let's find the momentum of Σ^-.

The time measured in S frame is $t_S = \gamma t_{S'} \Rightarrow \gamma = \frac{t_S}{t_{S'}} = \frac{L}{c t_{S'}}$

From Einstein's equation:

$$\mathcal{E}^2 = (mc^2)^2 + (pc)^2 \Rightarrow pc = \sqrt{\mathcal{E}^2 - (mc^2)^2} = mc^2\sqrt{\gamma^2 - 1} \Rightarrow$$

$$\boxed{p = mc\sqrt{\left(\frac{L}{ct_S'}\right)^2 - 1}} \qquad p = 1.197\,\frac{GeV}{c}\sqrt{\left(\frac{11.1}{3\times10^8 \cdot 1.48\times10^{-10}}\right)^2 - 1} \approx$$

$$\approx \boxed{299.25\ GeV/c}$$

(5) Griffith 3.15

(a) $p + p \to p + p + \pi^0$

The minimum energy needed for the reaction is

$$E = \frac{M^2 - 2m_p^2}{2m_p}c^2 = \frac{(2\times 938.27\,MeV + 134.98\,MeV)^2 - 2\times(938\,MeV)^2}{2\times 938\,MeV} =$$

$$= \boxed{1217.94\ MeV}$$

$$T = E - m_pc^2 = 1217.94\,MeV - 938.27\,MeV = \boxed{279.67\,MeV}$$

(b) $p + p \to p + p + \pi^+ + \pi^-$

According to the given equation

$$E = \frac{(2m_p + m_{\pi^+} + m_{\pi^-})^2 - 2m_p^2}{2m_p} = \frac{(2\times 938.27 + 2\times 139.57)^2 - 2(938.27)^2}{2\times 938.27} =$$

$$= \boxed{1538.07\ MeV}$$

$$T = E - m_p c^2 = 1538.07 - 938.27 = \boxed{599.80\ MeV}$$

(c) $\pi^- + p \to p + \bar{p} + n$

The threshold for this reaction is $E = \dfrac{(2m_p + m_n)^2 - m_p^2 - m_{\pi^-}^2}{2m_p} =$

$$= \frac{(2\times 938.27 + 939.57)^2 - 139^2 - 938.27^2}{2\times 938.27} = \boxed{3746.7\ MeV} \quad \boxed{T = E - m_{\pi^-} c^2 = 3607.1\ MeV}$$

(d) $\pi^- + p \to K^0 + \Sigma^0$

The threshold is $E = \dfrac{(m_{K^0} + m_{\Sigma^0})^2 - m_p^2 - m_\pi^2}{2m_p} = \dfrac{(497.67 + 1192.55)^2 - 139^2 - 938.27^2}{2\times 938.27} =$

$$= \boxed{1042.8\ MeV} \qquad T = E - m_{\pi^-} c^2 = 1042.8 - 139.6 = \boxed{903.2\ MeV}$$

(e) $p + p \to p + \Sigma^+ + K^0$

The threshold is $E = \dfrac{(m_p + m_{\Sigma^+} + m_{K^0})^2 - 2m_p^2}{2m_p} = \dfrac{(938.27 + 1189.37 + 497.67)^2 - 2m_p^2}{2\times 938.27} =$

$$= \boxed{2734.58\ MeV} \qquad \boxed{T = 1796.31\ MeV}$$

6 Griffiths 3.20

(a) The total four-momentum in the CM and in the lab is conserved.

$$(p_{tot})^2 = (p'_{tot})^2$$

(S)

$$p_1 = (p_x, 0, 0, \tfrac{E}{c})$$
$$p_2 = (0, 0, 0, mc)$$
$$p_{tot} = (p_x, 0, 0, \tfrac{T + 2mc^2}{c})$$

lab frame

(S')

$$p_1' = (p_x', 0, 0, E/c)$$
$$p_2' = (-p_x', 0, 0, E/c)$$
$$p'_{tot} = (0, 0, 0, 2E/c)$$

CM frame

Hence we have:

$$\frac{(T+2mc^2)^2}{c^2} - p_x{}^2 = \left(\frac{2E}{c}\right)^2$$

$$(\gamma mc^2 + mc^2)^2 - p_x{}^2c^2 = 4E^2 \Rightarrow E^2 = \frac{1}{4}\left[m^2c^4(1+\gamma)^2 - (\gamma mc^2)^2 + (mc^2)^2\right] =$$

$$= \frac{1}{4}\left[m^2c^4(1+\gamma)^2 - m^2c^4(\gamma^2+1)\right] = \frac{1}{4}m^2c^4\left[1+2\gamma+\gamma^2-\gamma^2+1\right] = \frac{\gamma+1}{2}m^2c^4$$

$$\boxed{E = mc^2\sqrt{\frac{\gamma+1}{2}}} \Rightarrow \gamma' = \sqrt{\frac{\gamma+1}{2}} = \frac{1}{\sqrt{1-v'^2/c^2}} \Rightarrow 1 - \frac{v'^2}{c^2} = \frac{2}{\gamma+1} \Rightarrow$$

$$\boxed{v' = c\sqrt{1-\frac{2}{\gamma+1}}}$$

(b) Hence the kinetic energy in CM frame is:

$$T' = \gamma' mc^2 - mc^2 \Rightarrow T' + mc^2 = \sqrt{\frac{1+\gamma}{2}}\, mc^2 \Rightarrow$$

$$(T')^2 + 2mc^2T' + m^2c^4 = mc^2(2mc^2+T)/2 \Rightarrow \boxed{T = 4T'\left(1+\frac{T'}{2mc^2}\right)}$$

7 Griffiths 3.22

$A + B \to C + D$ Mandelstam variables: $\boxed{\begin{array}{l} s \equiv (p_A+p_B)^2/c^2 \\ t \equiv (p_A-p_C)^2/c^2 \\ u \equiv (p_A-p_D)^2/c^2 \end{array}}$

(a) $s+t+u = \frac{1}{c^2}\left((\vec{p}_A+\vec{p}_B)^2 + (\vec{p}_A-\vec{p}_C{}^2) + (\vec{p}_A-\vec{p}_B)^2\right) =$

$= \frac{1}{c^2}\left[p_A^2 + 2\vec{p}_A\cdot\vec{p}_B + p_B^2 + p_A^2 - 2\vec{p}_A\cdot\vec{p}_C + p_C^2 + p_A^2 - 2\vec{p}_A\vec{p}_D + p_D^2\right] =$

Due to $\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$ we have:

$s+t+u = \frac{1}{c^2}\left[p_A^2 + 2\vec{p}_A\vec{p}_B + p_B^2 + p_A^2 - 2\vec{p}_A(\vec{p}_A+\vec{p}_B) + p_C^2 + p_A^2 + p_D^2\right] =$

$= \left[p_A^2 + p_B^2 + p_C^2 + p_D^2\right]/c^2 = \boxed{m_A^2 + m_B^2 + m_C^2 + m_D^2}$

(b) The CM energy of A could be found from the total four-momentum.

$(\vec{p}_A+\vec{p}_B)^2 = \left(\frac{E_A+E_B}{c}\right)^2$, due to $\sqrt{s} = \frac{E_1+E_2}{c^2}$ we have

$$sc^4 = E_A^2 + E_B^2 + 2E_AE_B = 2E_A^2 - m_A^2c^4 + m_B^2c^4 + 2E_AE_B$$

$$(s+m_A^2-m_B^2)c^4 = 2E_A^2 + 2E_AE_B = 2E_A(E_A+E_B) \Rightarrow$$

$$(s+m_A^2-m_B^2)c^2 = 2E_A(E_A+E_B)/c^2 = 2E_A\sqrt{s} \Rightarrow \boxed{E_A = \frac{(s+m_A^2-m_B^2)c^2}{2\sqrt{s}}}$$

(c) Let's find the lab energy of A if B is at rest.

The total four-momentum is conserved, hence

$$(\vec{p}_A + \vec{p}_B)^2 = sc^2 = \left(\frac{E_A}{c} + m_B c\right)^2 - |p_A|^2 = \left(\frac{E_A}{c}\right)^2 + 2E_A m_B + m_B^2 c^2 - p_A^2 \Rightarrow$$

$$sc^2 = m_A^2 c^2 + 2E_A m_B + m_B^2 c^2 \Rightarrow$$

$$\Rightarrow \boxed{E_A = \frac{(s - m_A^2 - m_B^2)c^2}{2m_B}}$$

(d) The total CM energy is $E'_{tot} = E'_A + E'_B$

$$sc^2 = (p_{tot})^2 = \frac{(E'_A + E'_B)^2}{c^2} \Rightarrow \boxed{E'_{tot} = \sqrt{s}\, c^2}$$