Solutions to the homework #5 (due Wednesday, Feb 25)

Problem 2 (Griffiths 1.12)

(a) The general formula for n flavors is $\boxed{\text{Combinations} = n^2}$

Hence, with $n=1$; there is only one meson $q\bar{q}$

$n=2$, there are 4 different mesons:

$\bar{q}_1 q_2$; $q_1 \bar{q}_2$; $\bar{q}_1 q_1$; $\bar{q}_2 q_2$

$n=3$, there are $3^2 = 9$ different mesons:

$\bar{q}_1 q_1$ $\bar{q}_2 q_2$ $\bar{q}_3 q_3$ $\bar{q}_1 q_2$ $\bar{q}_2 q_3$ $\bar{q}_1 q_3$ $q_1 \bar{q}_2$ $q_2 \bar{q}_3$ $q_1 \bar{q}_3$

$n=4$, there are 16 combinations

$n=5$, 25 combinations

$n=6$, there are 36 combinations

(b) For 5 quarks, the number of pairs of distinct mesons and anti-mesons is $\dfrac{n^2-n}{2} = \dfrac{25-5}{2} = \dfrac{20}{2} = \boxed{10}$

The number of self-conjugated meson states is $\boxed{n=5}$.

Problem 3 (Griffith 1.13)

A baryon consists of three quarks. Hence the general formula for n different quark flavors can be derived from the following argumentation:

- the number of combinations, where all quarks are the same (like $q_i q_i q_i$) is $\binom{n}{1} = \dfrac{n!}{(n-1)!\,1!}$
- the number of combinations with all different flavors (like $q_i q_j q_k$, where $i \neq j \neq k$) is $\binom{n}{3} = \dfrac{n!}{(n-3)!\,3!}$
- the number of combinations where 2 flavors are the same but the third one is different (like $q_i q_i q_j$, where $i \neq j$) is $2\binom{n}{2} = \dfrac{2n!}{(n-2)!\,2!} = \dfrac{n!}{(n-2)!}$,

As a result the number of baryon combinations is

$$\boxed{N = \binom{n}{1} + 2\binom{n}{2} + \binom{n}{3}} = \boxed{\frac{n(n+1)(n+2)}{6}}$$

for $n=1$: $N = \binom{1}{1} + 2\underset{\equiv\, 0}{\cancel{\binom{1}{2}}} + \underset{\equiv\, 0}{\cancel{\binom{1}{3}}} = \boxed{1}$ (qqq)

for $n=2$: $N = \binom{2}{1} + 2\binom{2}{2} + \underset{\equiv\, 0}{\cancel{\binom{2}{3}}} = 2 + 2 = \boxed{4}$ $\begin{pmatrix} q_1 q_1 q_1 \\ q_2 q_2 q_2 \\ q_1 q_1 q_2 \\ q_2 q_2 q_1 \end{pmatrix}$

for $n=3$: $N = \binom{3}{1} + 2\binom{3}{2} + \binom{3}{3} = 3 + 2\cdot 3 + 1 = \boxed{10}$

for $n=4$: $N = \binom{4}{1} + 2\binom{4}{2} + \binom{4}{3} = \frac{4!}{3!} + 2\,\frac{4!}{2!\,2!} + \frac{4!}{3!} = \boxed{20}$

for $n=5$: $N = \binom{5}{1} + 2\binom{5}{2} + \binom{5}{3} = \frac{5!}{4!} + 2\,\frac{5!}{3!\,2!} + \frac{5!}{3!\,2!} = \boxed{35}$

for $n=6$: $N = \binom{6}{1} + 2\binom{6}{2} + \binom{6}{3} = \frac{6!}{5!} + 2\,\frac{6!}{4!\,2} + \frac{6!}{3!\,3!} = 6 + 30 + 20 = \boxed{56}$

Problem 4 (Griffith 2.1)

The gravitational force is $F_g = G\,\frac{m_e^2}{r^2}$

The electrical force is $F_{el} = \frac{k_e\, e^2}{r^2}$

$\Rightarrow \boxed{\frac{F_g}{F_{el}} = \frac{G}{k_e}\left(\frac{m_e}{e}\right)^2}$

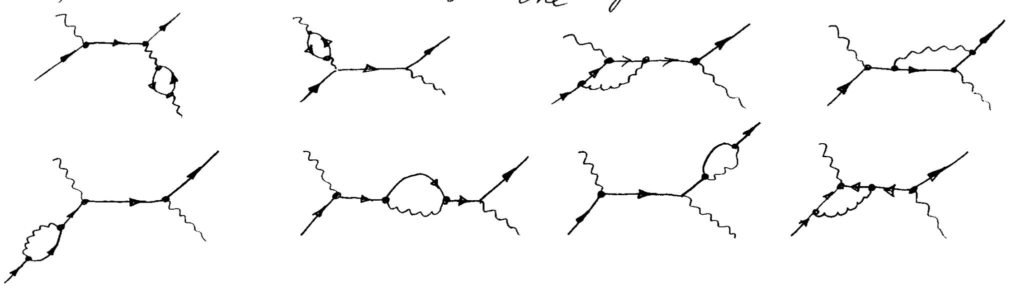
$$\frac{F_g}{F_{el}} = \frac{6.673\times 10^{-11}\ \frac{N}{m^2\cdot kg^2}}{8.988\times 10^{9}\ \frac{N\cdot m^2}{C^2}} \cdot \left[\frac{9.1\times 10^{-31}\,kg}{1.6\times 10^{-19}\,C}\right]^2 = \boxed{2.4\times 10^{-43}}$$

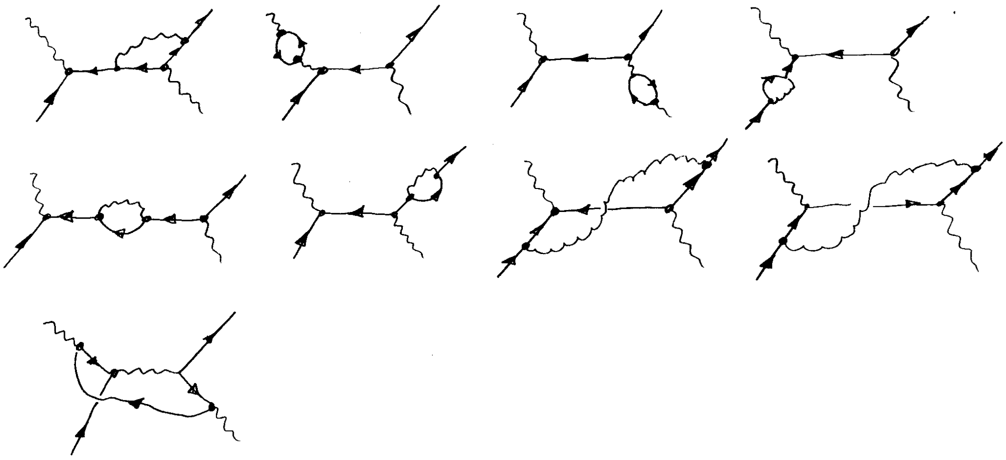
Problem 5 (Griffith 2.3)

The fourth-order diagrams for Compton scattering, $e^- + \gamma \to e^- + \gamma$

The principle for the construction is the following:

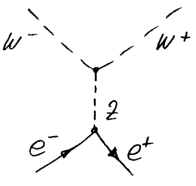
we have to connect four vertices with one electron went; one electron out and one γ line:



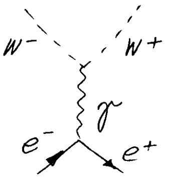


Problem 6

The lowest-order diagrams contributing to the process $e^+ + e^- \to W^+ + W^-$



The direct coupling of Z to W's



The coupling of γ to W's