Homework 3
(Due February 11)

Problem 1

The table of all the isotopes of Carbon:

the atomic weight, A	the atomic mass (amu)	the abundance %	the half life	the decay mode(s)
8	8.037		2×10^{-23} s	2p (^{6}Be)
9	9.031		126,5 ms	+ 2α (^{2}H)
10	10.016		19.225 s	e 10B
11	11.011		20.39 min	e 11B
12	12.000	98.89		
13	13.003	1.11		
14	14.003		5730 years	β (^{14}N)
15	15.010		2.449 s	β (^{15}N)
16	16.015		0.747 s	β (^{16}N)
17	17.023		193 ms	β (^{17}N) β+n (^{16}N)
18	18.027		95 ms	β (^{18}N)
19	19.035		49 ms	β (^{19}N)
20	20.040		14 ms	β (^{20}N)
21	21.050			
22	22.057		> 200 ns	

The table of all isotopes of Iron:

45	45.014		> 350 nm	EC (^{46}Mn)
46	46		20 ms	EC (^{47}Mn)
47	46.992		27 ms	EC (^{48}Mn)
48	47.98		44 ms	EC (^{49}Mn)
49	48.973		70 ms	EC (^{50}Mn)
50	49.962		150 ms	EC (^{51}Mn)
51	50.956		305 ms	EC (^{52}Mn)
52	51.948		8,275 h, 45.9 s	EC (^{53}Mn)
53	52.945			
54	53.939	5.85		
55	54.9382		2.73 years	(EC ^{55}Mn)
56	55.934	91.75		
57	56.935	2.12		
58	57.933	0.28		
59	58.934		44.503 days	β (^{59}Co)
60	59.934		1.5×10^{6} years	β (^{60}Co)
61	60.936		5.98 min	β (^{61}Co)
62	61.936		68 s	β (^{62}Co)
63	62.94		6.1 s	β (^{63}Co)
64	63.94		2 s	β (^{64}Co)
65	64.944		0.4 s	β (^{65}Co)
66	65.945		600 ms	β (^{66}Co)
67	66.95		> 200 ns	β (^{67}Co)
68	67.952		0.1 s	β (^{68}Co)
69	68.957		> 150 s	β

Problem 2

$T_p = 10.8$ MeV, Elastic scattering from ^{197}Au

(a) No, we don't relativistic kinematics due to $M_p \approx 938$ MeV $>> T_p$

(b) The maxim. angle is π.

(c) At $\theta = \pi$ the momentum of the proton can be found from conservation law of energy and momentum:
$$\begin{cases} T_p = \dfrac{M_p v_p^2}{2} + \dfrac{m_{Au} v_{Au}^2}{2} \\ M_p v_0 = m_{Au} v_{Au} - M_p v_p \end{cases}$$
due to $m_{Au} >> M_p$

(d) $p_{Au} = 0$; $P_p \simeq 2p_0 = 2M_p v_{p_0}$
$= \boxed{285 \text{ MeV}}$

$T_e = 10.8$ MeV, electrons are elastically scattered from ^{197}Au.

(a) Yes $m_e = 0.511$ MeV $\ll T_e$

(b) The maximum angle is π.

(c) $T_e = \dfrac{m_e c^2}{\sqrt{1 - v^2/c^2}} - mc^2 \Rightarrow (T_e + m_e c^2)^2 = m_e^2 c^4 \left[1 - v^2/c^2\right] \Rightarrow$

$$\Rightarrow \boxed{v = c\sqrt{1 - \left[\frac{T_e}{mc^2} + 1\right]^{-2}}}$$

$(T_e + mc^2)^2 = p^2c^2 + m^2c^4 \Rightarrow pc = \sqrt{(T_e + mc^2)^2 - m^2c^4} = \sqrt{(10.8 + 0.511)^2 - 0.}$
$= 11.3$ MeV

(d) Because of a huge difference in masses $M_{Au} >> m_e \Rightarrow \boxed{p_{Au} = 0}$
$p_e = 2p_0 = \boxed{22.3 \text{ MeV.}}$

Problem 3

(a) The form factor for a spherically symmetric nuclear charge density
$$F(q^2) = \int e^{\frac{i\vec{q}\vec{r}}{\hbar}} f(\vec{r}) r^2 dr d\Omega = 2\pi \int\int_0^\pi e^{i\, qr\cos\theta/\hbar} \cdot f(\vec{r}) r^2 dr\, d\cos\theta$$
$$= 2\pi \int_0^\infty \int_{-1}^1 e^{iqrx/\hbar} f(r) r^2 dr\, dx =$$
$$= 2\pi \int_0^\infty f(r) r^2 dr \cdot \left. \frac{e^{iqrx/\hbar}}{iqr/\hbar} \right|_{-1}^{1} = \frac{4\pi\hbar}{q} \int_0^\infty r\,dr\, f(\vec{r}) \sin\left(\frac{qr}{\hbar}\right) dr$$

Due to the spherical symmetry $f(\vec{r}) = f(r) = \dfrac{\rho(r)}{Ze}$, hence
$$\boxed{F(q^2) = \frac{4\pi\hbar}{qZe} \int \rho(r) \cdot r \cdot \sin\left(\frac{qr}{\hbar}\right) dr}$$

(b) Using expansion $\sin\alpha = 0 + f'(0)\,\alpha + \frac{f''(0)}{2!}\cdot\alpha^2 + \frac{f'''(0)}{3!}\alpha^3 + \dots$
and for $\frac{qr}{\hbar} = n\cdot\pi$ the integral is zero,
the only function will be $\sin\left(\frac{qr}{\hbar}\right) \approx \frac{qr}{\hbar} - \left(\frac{qr}{\hbar}\right)^3\frac{1}{6} + \dots$
Let's substitute that into the integral $F(q^2)$:

$$F(q^2) \approx \frac{4\pi\hbar}{qZe}\left[\int_0^\infty \rho(r)\cdot r\,\frac{qr}{\hbar}\,dr - \int \rho(r)\cdot\frac{r}{6}\left(\frac{qr}{\hbar}\right)^3 + \dots\right] =$$

$$= \int_0^\infty 4\pi\, f(r)\cdot r^2 dr - \frac{1}{6}\cdot\int_0^\infty f(r)\cdot q^2\cdot\frac{r^4}{\hbar^4}\,4\pi\,dr = \boxed{1 - \frac{1}{6}\frac{q^2}{\hbar^2}\langle r^2\rangle + \dots}$$

Problem 4

Notation $nL_J = 1S_{1/2}$ The nuclei are in their ground state. 82 neutrons + One extra neutron. New state?

Using the Shell Model, "82 neutrons" is a magic number, because neutrons in a nucleus are in filled shells. According to the plot from the lecture (or the picture below) the next state above "82" is $\boxed{1h_{7/2}}$