Homework #2

Problem 1

Due to the ratio of the stragths of the strong and electromagnetic forces the same, and $F_e \sim Z^2\alpha^2$ and a strong force is the same $\Rightarrow$ $\boxed{Z\alpha' = Z_0\alpha}$

- for ${}^{186}_{94}Pu$: $(74)\alpha = 94\alpha' \Rightarrow \alpha' = \frac{74}{94}\alpha = \boxed{0.79\alpha = \frac{1}{174}}$
- for ${}^{186}_{88}Rg$: $88\alpha' = 74\alpha \Rightarrow \alpha' = \frac{74}{88}\alpha = \boxed{0.84\alpha \approx \frac{1}{163}}$
- for ${}^{186}_{82}Pb$: $74\alpha = 82\alpha' \Rightarrow \alpha' = \frac{74}{82}\alpha \approx \boxed{0.90\alpha \approx \frac{1}{152}}$

Problem 2

Let's estimate the value of α_c with simple electrostatic. The potential energy of a uniformly charged sphere is

$$U_c = \frac{\varepsilon_0}{2}\int E^2 d\tau = \frac{\varepsilon_0}{2}\left[\frac{1}{4\pi\varepsilon_0}\right]^2 q^2\left[\int_R^\infty \frac{4\pi r^2 dr}{r^4} + \int_0^R 4\pi r^2 dr\cdot\left(\frac{r}{R^3}\right)^2\right] =$$

$$= \frac{q^2}{8\pi\varepsilon_0}\left[\int_R^\infty \frac{dr}{r^2} + \int_0^R \frac{1}{R}r^3 dr\right] = \boxed{\frac{3q^2}{20\pi\varepsilon_0 R}}$$

The binding energy of ${}^{184}_{74}W$ is $1.473\ MeV = B(184, 74)$

According to the Liquid Drop Model $B(Z,A) = a_v A - a_s A^{2/3} - a_c\frac{Z^2}{A^{1/3}} - a_a\frac{(A-2Z)^2}{A} + \frac{\delta}{\sqrt{A}}$

From the lecture: $a_v \approx 15.67\ MeV$
$a_s = 17.23\ MeV$
$a_a = 23.3\ MeV$

$$U_c = a_c\frac{Z^2}{A^{1/3}} = a_v A - a_s A^{2/3} - a_a\frac{(A-2Z)^2}{A} - B(Z,A) + \frac{\delta}{\sqrt{A}}$$

$$a_c = \frac{a_v A^{4/3}}{Z^2} - a_s\frac{A}{Z^2} - a_a\frac{(A-2Z)^2}{A^{2/3}Z^2} - \frac{B(Z,A)\cdot A^{1/3}}{Z^2} + \frac{\delta A^{1/3}}{A^{1/2}Z^2}$$

$$a_c = \frac{15.67\times(184)^{4/3}}{74^2} - 17.23\cdot\frac{184}{74^2} - 23.3\frac{(184-2\cdot74)^2}{(184)^{2/3}\cdot74^2} - \frac{1.473\cdot(184)^{1/3}}{74^2} \approx 0.7\ MeV$$

Or from equation for $U_c = \frac{3q^2}{20\pi\varepsilon_0 R}$, $R = 1.25\sqrt[3]{A}$ $\Rightarrow$ $U_c = \frac{3Z^2e^2}{20\pi\varepsilon_0 \cdot 1.25 A}$

$$U_c = \frac{3e^2}{20\pi\varepsilon_0 \cdot 1.25}\frac{Z^2}{A^{1/3}} = a_c \frac{Z^2}{A^{1/3}} \Rightarrow$$

$$a_c = \frac{3e^2}{20\pi\varepsilon_0 \cdot 1.25} \approx \frac{3}{1.25\times 5}\left(\frac{e^2}{4\pi\varepsilon_0}\right) = \frac{3\times(1.6\times10^{-19})^2\times 9\times 10^9}{1.25\times 5\times(1.6\times10^{-19})} = \boxed{0.69\ MeV}$$

Problem 3

$$^{238}Pu \rightarrow {}^{234}U + 5.49\ MeV$$

$\tau = 127$ years $\quad \tau = 3.5\times10^5$ years

(a) The power 400W, a power conversion efficiency 6.2%. Let's find the mass of plutonium needed.

$$\frac{dN}{dt} = -\lambda N, \text{ where } \lambda = \frac{1}{\tau} = \frac{1}{127\times365\times24\times3600} = 2.5\times10^{-10}\,s^{-1}$$

The space probe travels during 4 years 6 days $= 13.2\times10^7\,s$

The power needed $P = 400W \times 0.062^{-1} = 6451.6\ J/s = 4.03\times10^{16}\ \frac{MeV}{s}$

The number of nuclei of Pu needed:

$$\frac{dN}{dt} = \frac{P}{P_0} = \frac{4.03\times10^{16}\ MeV/s}{5.49\ MeV} = 7.3\times10^{15}\ \frac{decay}{s}$$

Hence the number is $N = \frac{dN}{dt}\cdot\tau = \frac{7.3\times10^{15}}{2.5\times10^{10}} \approx 2.92\times10^{25}$

So the initial number of nuclei needed is $N_0 = Ne^{\lambda t} =$

$$= 2.9\times10^{25}\times e^{13.2\times10^7\times2.5\times10^{-10}} \approx \boxed{3.02\times10^{25}}$$

The mass of Pu : $m_{Pu} = 3\times10^{25}\times238\times1.66\times10^{-27}\,kg \approx \boxed{12.2\,kg}$

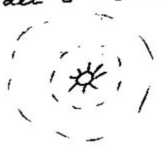
(b) The electric power, which was available at 30.1 AU at $t \approx 12$ years:

$$N = N_0 e^{-\lambda t} = 3\times10^{25} e^{-2.5\times10^{-10}\times12\times365\times24\times3600} \approx 2.75\times10^{25}$$

The decay rate at the moment $R(12\,years) = \lambda N(12\,years) = 2.75\times10^{25}\times2.5\times10^{-10} =$
$= 6.9\times10^{+15}\ \frac{decay}{s}$ $\Rightarrow$ The power $P(12\,years) = 6.9\times10^{15}\times5.49\frac{MeV}{s} =$
$= 3.79\times10^{16}\frac{MeV}{s} = 6064\,W$, but the available power $P_{avail} = 0.062\cdot P = 375\,W$

$$\boxed{P_{avail} = 375\,W}$$

(c) Let's estimate the area of solar cell: The intensity $\sim \frac{1}{r^2}$:



$$I(R_{orb}) = \frac{\text{Power of the sun}}{4\pi R_{orbit}^2} \cdot \text{Area}$$

At 1 Au the power is $\frac{10.5\times10^3 W}{730 m^2} = 0.1497 \frac{W}{m^2}$

If 400W is needed then area $= \frac{400W}{0.1497\, W/m^2}\cdot\left(\frac{9.5\, AU}{1\, AU}\right)^2 = \boxed{2509 m^2}$

Problem 4 (Griffiths 1.3)

$E^2 = m^2c^4 + \vec{p}^2c^2$, the minimum momentum can be found from $\Delta x\cdot\Delta p \geq \hbar$.

$$\Delta p \geq \frac{\hbar}{10^{-15}m} = \frac{\hbar c}{1\cdot fm\cdot c} = \frac{200\, MeV\cdot \cancel{fm}}{c\cdot \cancel{fm}} = \frac{200\, MeV}{c}$$

Hence the minimum energy $E \geq \sqrt{m^2c^4 + p^2c^2} = \sqrt{(0.511\, MeV)^2 + 400 (MeV)^2} \approx$

$$\boxed{E_{min} = 200\, MeV}$$

From Fig 1.6 the energy are much smaller ($\sim 1 \div 12$ keV).

Problem 5

$\theta_{min} = 1.22\frac{\lambda}{D}$. α-particles have energy $E_{kin} = 130$ MeV, they are scattered of ^{59}Co nucleus.

For each Co nucleus $D = 2R_{Co} = 2\times 1.25\times A^{1/3} = 2\times1.25\times(59)^{1/3} = 9.7$ fm

The energy $E^2 = (E_{kin} + mc^2)^2 = p^2c^2 + m^2c^4 \Rightarrow$

$$pc = \sqrt{(E_{kin}+mc^2)^2 - m^2c^4} = \sqrt{(130+3727)^2 - 3727^2} = 993\, MeV$$

The wavelength $\lambda = \frac{\hbar}{p} = \frac{197.3\, MeV\cdot fm}{2\pi\cdot 993\, MeV} = 0.032$ fm

Finally $\theta_{min} = 1.22\,\frac{0.032\, fm}{9.7\, fm} = 0.00793\, rad = \boxed{0.45^\circ}$