Homework #1 Solutions

Problem 1

The number of α-particles per second hitting the detector is

$\boxed{N_\alpha = L \cdot \Delta\Omega \cdot \frac{d\sigma}{d\Omega}}$ (*), where luminosity $L = \Phi_{in} \cdot N_t$, where N_t is a rate of hitting α-particles.

where $N_t = n_t A \cdot d$

density of a target

First, let's calculate density of target n_t:

$$n_t = \rho \times \left(\frac{1}{196.97\,amu}\right)\left(\frac{1\,amu}{1.66\times10^{-27}kg}\right) = 19.3\frac{g}{cm^3}\left(\frac{1}{196.97}\right)\left(\frac{1}{1.66\times10^{-24}g}\right) =$$

$$= 5.9\times10^{22}\,cm^{-3} = 5.9\times10^{28}\,m^{-3}$$

$$\boxed{n_t = 5.9\times10^{28}\,m^{-3}}$$

Hence a rate of hitting α-particles N_t:

$$N_t = n_t A \cdot d = 5.9\times10^{28}\,m^{-3}\times(10^{-3}m)^2\times 2.6\times10^{-6}\,m = 1.53\times10^{17}$$

Finally the luminosity $L = \Phi_{in} \cdot N_t$ is

$$L = 3\times10^{6}\,\frac{1}{cm^2\cdot s}\times1.53\times10^{17} = \boxed{4.6\times10^{23}\,\frac{1}{cm^2\cdot s}}$$

For the equation (*) we need to find the differential cross section: For Rutherford scattering

$$\frac{d\sigma}{d\Omega} = \frac{(k_e z Z' e^2)^2}{(4E_t)^2 \sin^4\left(\frac{\theta}{2}\right)}$$ → This result was derived on the lecture.

Where z is a charge multiplier of α particles, $z=2$
Z' is a charge multiplier of the target, for Au $Z'=79$

Let's substitute these numbers into the differential cross section and switch to fermi units:

$$\frac{d\sigma}{d\Omega} = \frac{4\times 79^2}{(4E_{kin})^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} = \frac{24964}{4\times 8.4\,\text{MeV}}\left(\frac{e^2}{4\pi\varepsilon_0\times\hbar c}\right)^2 \frac{(\hbar c)^2}{\sin^4(\theta/2)} =$$

$$= \frac{24964}{1128.96\,(\text{MeV})^2}\left(\frac{1}{137}\right)^2 \frac{(197\,\text{MeV}\cdot\text{fm})^2}{\sin^4\left(\frac{\theta}{2}\right)} = \boxed{\frac{45.72}{\sin^4(\theta/2)}\ (\text{fm})^2}$$

As a result we have

$$N_\alpha = L\cdot\Delta\Omega\cdot\frac{d\sigma}{d\Omega} = 4.6\times 10^{23}\,\frac{1}{\text{cm}^2\cdot\text{s}}\times 0.001\times\frac{45.72\ \text{fm}^2}{\sin^4(\theta/2)} =$$

$$= \frac{4.6\times 10^{23}\times 0.001\times 45.72\cdot\text{fm}^2}{(10^{13}\text{fm})^2\cdot\text{s}\quad \sin^4(\theta/2)} = \boxed{\frac{2.1\times 10^{-4}}{\sin^4(\theta/2)}\quad (\text{s}^{-1})}$$

(a)

$\theta,^\circ$	$\sin\left(\frac{\theta}{2}\right)$	$N_\alpha(\theta) = \frac{2.1\times 10^{-4}}{\sin^4(\theta/2)}, \text{s}^{-1}$
5	0.04361	58
10	0.08716	3.6
30	0.25882	0.047
60	0.50000	0.0034
90	0.70711	0.00084
120	0.86603	0.00037

(b) See plot attached

Problem 2

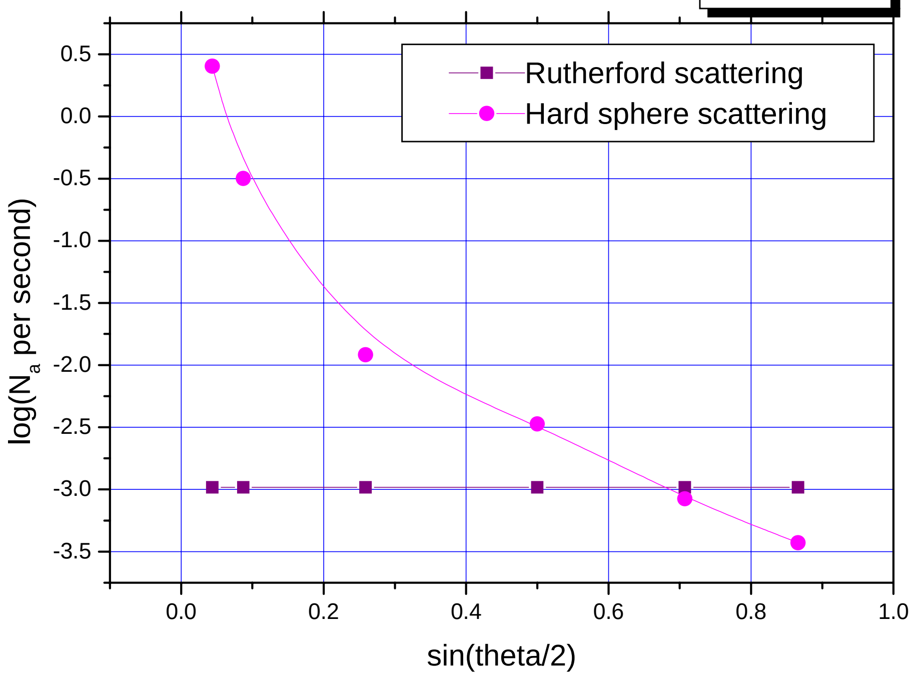
If we replace each gold nucleus by a hard sphere with radius $R = 9.5\times 10^{-15}\,\text{m} = 9.5\ \text{fm}$, we will have different differential cross section.

From the lecture $\left.\frac{d\sigma}{d\Omega}\right|_{\text{Hard sprere}} = \frac{R^2}{4} = \frac{(9.5\ \text{fm})^2}{4} = 22.56\ \text{fm}^2$

As a result : $N_\alpha = 0.001\times\frac{46\times 10^{23}}{(10^{13}\text{fm})^2\cdot\text{s}}\times 22.56\ \text{fm}^2 = \boxed{0.00104\quad \text{s}^{-1}}$

$\boxed{N_\alpha \text{ is a Constant !}}$

Problem 1&2



Problem 3

Let's transfer horizontal axis from θ to $p^2 = 4(mv)^2 \sin^2\left(\frac{\theta}{2}\right)$

The kinetic energy given for α-particles:

$$E_{kin} = E - m_\alpha c^2 = \sqrt{p^2c^2 + (m_\alpha c^2)^2} - m_\alpha c^2$$

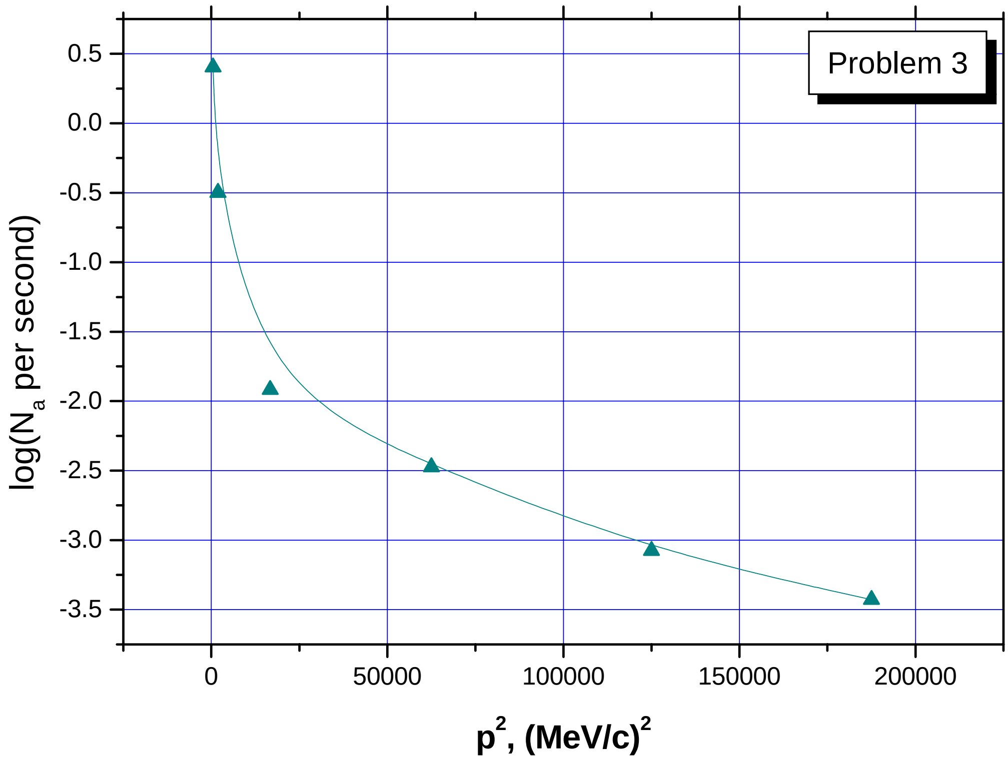
But due to $m_\alpha \gg p/c$, we can apply $E_{kin} = \frac{mv^2}{2} \Rightarrow$

$$\Rightarrow (mv)^2 = m \cdot (mv^2) = 2mE_{kin} \Rightarrow \boxed{p^2 = 8m_\alpha E_{kin} \sin^2(\theta/2)}$$

See plot attached.

$$m_\alpha = 4 \times 931.5 \ \frac{MeV}{c^2} \quad , \quad E_{kin} = 8.4 \ MeV$$

$$p^2 = 8 \times 4 \times 931.5 \ \frac{MeV}{c^2} \times 8.4 \ MeV \sin^2\left(\frac{\theta}{2}\right) = \boxed{2.5 \times 10^5 \sin^2\left(\frac{\theta}{2}\right) \ \left(\frac{MeV}{c}\right)^2}$$



Problem 3
log(N_a per second)
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
0
50000
100000
150000
200000
p^2, (MeV/c)^2