Physics 618 Twenty-seventh Lecture April 28, 2017

Last time we discussed the positive-mass representations of the Poincaré group and the corresponding finite representations of the Lorentz subgroup, including Weyl (left or right-handed) spinors and the Dirac spinor. Hoping to find a nontrivial incorporation of internal symmetries (such as isospin, SU(3) color or flavor, or electroweak SU(2)×U(1)) with the Lorentz symmetry, we were first blocked by the Coleman-Mandula theorem but then encouraged by introducing a symmetry larger than a Lie group, namely a **supersymmetry**.

While the elements of a Lie group can be written as exponentials of Lie algebra generators multiplied by real numbers, the elements of supersymmetry are formal exponentials of a superalgebra with coefficients, some of which are anticommuting rather than commuting objects (and are also spinors under Lorentz transformtions). We gave a simple example consisting of two noninteracting scalars and one Majorana spinor, with a formal variation of the fields in which bosons mix with fermions, but the action is unchanged (except for irrelevant surface terms). We learned that the greatest possibility contains

- a set of Majorana generators Q_{ja}, where a is a spinor index and j an integer $1 \leq j \leq N$ which, we shall see, is limited to $N \leq 8$.
- the Poincaré generators P^μ and $L_{\mu\nu}$
- a set of generators of an internal symmetry B_m
- a set of "central charges" Z_{jk} which commute with everything, are Lorentz scalars, but may emerge from the anticommutator of two Q's.

The supersymmetry generators Q obey anticommutation relations with each other, but commutation relations with the other (bosonic) generators, and in particular transform as a hermitean N dimensional representation of the internal Lie group generated by the B's, but as Majorana spinors under the Lorentz group.

Today

We will first examine what we might have for an irreducible representation for a single particle, in particular for a massless one, and the implication of the field-theoretic restriction that restricts the helicity λ to be ≤ 2. Of course the graviton does have $|\lambda| = 2$, and this restriction will tell us that $N \leq 8$. We will examine the content of the $N = 8$ representation including the graviton, and compare to what we need in a complete theory of everything, including gravity, strong, weak and electromagnetic interactions. Unfortunately, while we come close, we don't quite make it.

Then we will turn to reexpressing the fields of a supersymmetric theory in terms of a superfield. We will see how the supersymmetry generators are analogous to momenta, but, because they don't commute (or anticommute), their action is rather more intricate.

This is the final lecture, and so we will not get to conformal invariance. Sorry about that.

Reminder:

> The Final exam is on May 4 at noon (to 3:00 PM) in SEC room 206. It will be closed book, but you may bring in 3 sheets of handwritten notes.