Physics 618 Homework #1
Due: Jan. 26, 2017 at 4:00 PM sharp!

1 Find the matrix group generated by the matrices $i\sigma_x$ and $i\sigma_y$, where the σ's are the ordinary Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Show that the group has order 8 and has five conjugacy classes, but that it is not isomorphic to D_4. Hint: how many elements of order 4 are there?

2 Construct the multiplication table of the permutation group on three objects, S_3, and obtain its conjugacy classes.

3 We defined conjugacy of two elements of a group G by saying that B is conjugate to C, $(B \cong C)$, if there exists another element $A \in G$ such that $A^{-1}BA = C$. Prove formally that congugacy is an equivalence relation, *i.e.*
a) Conjugacy is reflexive: $A \cong A$ for all $A \in G$.
b) Conjugacy is symmetric: if $A \cong B$ then $B \cong A$.
c) Conjugacy is transitive: If $A \cong B$ and $B \cong C$, then $A \cong C$.