Kinematics of p. 218

The cut begins at

$$f(x) = (1-x)m_0^2 + x\mu^2 - x(1-x)p^2 = 0$$

which, by the quadratic formula, with $a = p^2$, $b = \mu^2 - p^2 - m_0^2$, $c = m_0^2$, gives

$$\begin{aligned} x &= \frac{1}{2} + \frac{m_0^2 - \mu^2}{2p^2} \pm \frac{\sqrt{[p^2 + m_0^2 - \mu^2]^2 - 4p^2 m_0^2}}{2p^2} \\ &= \frac{1}{2} + \frac{m_0^2 - \mu^2}{2p^2} \pm \frac{1}{2p^2}\sqrt{[p^2 - (m_0+\mu)^2][p^2 - (m_0-\mu)^2]} \end{aligned}$$

has solutions as given in 7.20, with k as given at the top of 219. In the center of mass, a pair of particles with $\pm\vec{k}$ and masses m_0 and μ has energy

$$\sqrt{p^2} = \sqrt{k^2 + m_0^2} + \sqrt{k^2 + \mu^2}.$$

Squaring,

$$p^2 = 2k^2 + m_0^2 + \mu^2 + 2\sqrt{k^2 + m_0^2}\sqrt{k^2 + \mu^2},$$

so

$$\begin{aligned} &(p^2 - 2k^2 - m_0^2 - \mu^2)^2 = 4\left(k^2 + m_0^2\right)\left(k^2 + \mu^2\right) \\ &p^4 + 4k^4 - 4k^2(p^2 - m_0^2 - \mu^2) - 2p^2(m_0^2 + \mu^2) + (m_0^2 + \mu^2)^2 \\ &\quad = 4k^4 + 4k^2(m_0^2 + \mu^2) + 4m_0^2\mu^2 \\ 4p^2k^2 &= p^4 - 2p^2(m_0^2 + \mu^2) + (m_0^2 - \mu^2)^2 \\ &= \left(p^2 - (m_0+\mu)^2\right)\left(p^2 - (m_0-\mu)^2\right) \end{aligned}$$

in agreement with the equation at the top of pate 219:

$$k = \frac{1}{2\sqrt{p^2}}\sqrt{[p^2 - (m_0+\mu)^2]\,[p^2 - (m_0-\mu)^2]}.$$

As $f(0) > 0$ and $f(1) > 0$, Σ_2 has a cut only if $f(x)$ vanishes at two points in $(0,1)$. For large positive p^2, the roots are at $\frac{1}{2}(1 \pm (1 - \mu^2/p^2)) \in (0,1)$, so there is a cut, and as p^2 diminishes, the zeros of f approach each other but the cut remains until the square root vanishes at $\sqrt{p^2} = m_0 + \mu$. For $m_0 - \mu < \sqrt{p^2} < m_0 + \mu$ the square root is imaginary so f does not change sign in $(0,1)$ and there is no cut. For large negative p^2 the zeros are at $x = \frac{1}{2}(1 \pm (1 + \mu^2/|p^2|)) \notin (0,1)$, and the zeros cannot cross 0 and 1, so f remains positive and there is no cut for $p^2 < (m_0+\mu)^2$, showing that the cut in Σ_2 is only from energies of multiparticle states.