

Lecture 16

Oct. 28, 2013

$$e^+ e^- \rightarrow \mu^+ \mu^-; \quad R; \quad e \mu \text{ Scattering}$$

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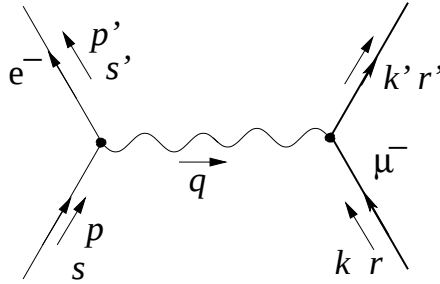
We now have the required rules for calculating real QED processes, at least until we run into difficulties at higher orders of perturbation theory, though we still have some calculational tricks to learn.

We have seen that the electron is described by a Dirac field ψ and the Lagrangian density for photons and electrons is

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m_e) \psi - e \bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

The μ lepton behaves just like the electron, except for having a higher mass m_μ , so if we wish to describe μ 's as well as electrons, we need to add a new field, which I will call Ψ , and add $\bar{\Psi} (i \not{\partial} - m_\mu) \Psi - e \bar{\Psi} \gamma^\nu \Psi A_\nu$ to $\mathcal{L}$. The propagator for the muon is the same as for the electron except for the value of the mass, and the charge and the interaction terms with photons the same.

We considered the scattering of nonidentical fermions, at least at low energy. The diagram for electron μ^- scattering is just like what we considered for the Yukawa case, except the scalar particle is replaced by a photon (usually shown as a sine wave), and we ought to distinguish m_e from m_μ



We found that the invariant amplitude was given by

$$i\mathcal{M} = (-ie)^2 \bar{u}_e^{s'}(p') \gamma^\mu u_e^s(p) \frac{-ig_{\mu\nu}}{(p' - p)^2} \bar{u}_\mu^{r'}(k') \gamma^\nu u_\mu^r(k).$$

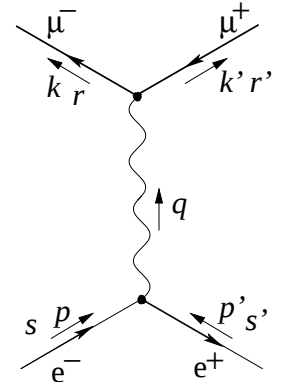
Then there is a related process, which is the conversion of one kind of fermion-antifermion pair into another — look at our last diagram sideways! Let one be electron-positron colliding beams, and consider an outgoing pair of fermions, perhaps $\mu^+ - \mu^-$.

Today we will consider this process, to get the invariant amplitude

$$\mathcal{M}(e^-, p, s + e^+, p', s' \rightarrow \mu^-, k, r + \mu^+, k', r')$$

to second order in the electric charge e . Then we will discuss calculational tricks which simplify the answer in the common situation that we have unpolarized beams and don't detect the final spins. We will discuss the angular distribution but more interestingly the total cross section as a method of detecting new particles, and color.

The amplitude is given by



$$\begin{aligned} & {}_0 \langle k, r; k' r' | (-ie) \int d^4 x \bar{\Psi}(x) \gamma^\rho \Psi(x) A_\rho(x) \\ & \quad \times (-ie) \int d^4 y \bar{\psi}(y) \gamma^\sigma \psi(y) A_\sigma(x) | p s; p' s' \rangle_0 \\ & = 4 \sqrt{E_p E_{p'} E_k E_{k'}} \\ & \quad \times \langle 0 | \overbrace{b_{k'}^{r'} a_k^r} (-ie) \int d^4 x \bar{\Psi}(x) \gamma^\rho \Psi(x) A_\rho(x) \\ & \quad \times (-ie) \int d^4 y \overbrace{\bar{\psi}(y) \gamma^\sigma \psi(y) A_\sigma(x)} a_p^{\dagger s} b_{p'}^{\dagger s'} | 0 \rangle \\ & = \bar{v}_e^{s'}(p') (-ie \gamma^\sigma) u_e^s(p) \left(\frac{-ig_{\rho\sigma}}{q^2} \right) \bar{u}_\mu^r(k) (-ie \gamma^\rho) v_\mu^{r'}(k'). \end{aligned}$$

Read Peskin and Schroeder pp. 131–141, 153–158

Note: On page 140, the book misdefines R , which should be the *ratio* of the cross section for $e^+e^- \rightarrow \text{hadrons}$ to the value we just calculated for $e^+e^- \rightarrow \mu^+\mu^-$, rather than the value of the cross section for $e^+e^- \rightarrow \mu^+\mu^-$. Note it is defined correctly in Figure 5.3.