Lecture 12 Oct. 14, 2013

Wick's Theorem and Feynman Diagrams

Last time we found that we could express any vacuum expectation value of the Time-ordered product of the full, interacting fields as

$$\langle\Omega|\, T\left(\prod_j \phi(x_j)\right)|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|\, T\left(\prod_j \phi_I(x_j)\right) e^{-i\int_{-T}^{T} dt\, H_I(t) dt}\, |0\rangle}{\langle 0|\, T e^{-i\int_{-T}^{T} dt\, H_I(t) dt}\, |0\rangle}$$

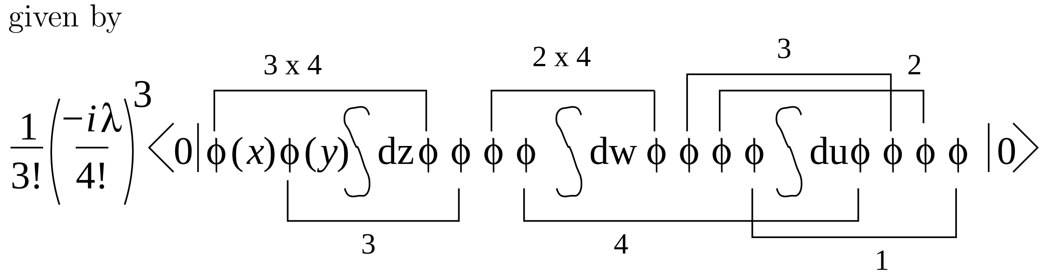
Read Peskin and Schroeder, pages 88-99.

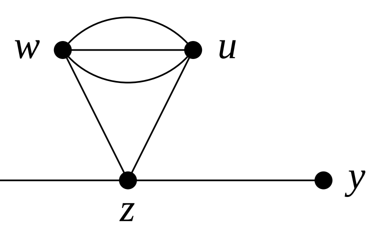
On page 92:

In order to do something different, let's consider a different piece in the contraction of

$$\langle 0|\, \phi(x)\phi(y)\frac{1}{3!}(-i\mathcal{H}_I(z))(-i\mathcal{H}_I(w))(-i\mathcal{H}_I(u))\, |0\rangle$$

given by





For the contraction with $\phi(x)$, we get the same diagram if we contract with any of the four ϕ's in any of the three internal vertices[1], for a factor of 12. Call that vertex the z vertex. Then $\phi(y)$ can be contracted with any of the three remaining ϕ's on the z vertex[2] So we multiply by 3.

[1] The other possibility, contracting with $\phi(y)$, would give a disconnected diagram (see p. 96) consisting of a noninteracting propagator times a product of disconnected pieces.

[2] Of course it could also be contracted with a different vertex, but that would not give rise to the same Feynman diagram. There are 8 equivalent ϕ's for that possibility. Then there are several distinct ways the next ϕ on the z vertex can be contracted, several of which lead to the diagram explored in the book on p. 92.

Then the first of the remaining two ϕ's from the z vertex can be contracted with any of the four ϕ's in either of the remaining internal vertices[3], so this gives us a factor of 8, and we choose to call the attached vertex w. The last ϕ on z must then be contracted with the remaining unattached vertex[4], u to any of the 4 ϕ's. There remains the three ϕ's on the w vertex which can be attached to the three remaining ϕ's on the u vertex in 3! ways[5]. So the symmetry factor for this diagram is

$$S = \frac{12 \times 3 \times 8 \times 4 \times 3!}{3! \times (4!)^3} = \frac{1}{12}.$$

[3]Or it could be contracted with the other z ϕ, which would give the first graph on page 93 multiplied by vacuum bubbles.

[4]Or to another of the w ϕ's, in which case we could get other graphs not shown in the book.

[5]Or perhaps there is only one w–u connection, in which case the diagram looks like a mouse.