Some Comments on P&S treatment of BRST

Below 16.49

That Q^2 vanishes on B and on $\bar{c}$ is trivial, the first killed by the first Q and the second converted by the first Q into a B and then killed by the second Q. For the fermion field

$$\begin{aligned} Q^2\psi = Q(igc^a t^a \psi) &= igQ(c^a)t^a\psi - igc^a t^a Q(\psi) \\ &= ig\left\{-\frac{1}{2}gf^{abc}c^b c^c t^a\psi - igc^a t^a c^b t^b \psi\right\} \\ &= -\frac{1}{2}g^2\left\{if^{abc}c^b c^c t^a\psi - c^a c^b\left[t^a, t^b\right]\psi\right\} \\ &= -\frac{1}{2}g^2\left\{if^{abc}c^b c^c t^a\psi - if^{abc}c^a c^b t^c\psi\right\} = 0. \end{aligned}$$

It is important to notice in the first line, that when the anticommuting Q passes through the grassman c^a, we pick up a minus sign. (I missed that in lecture).

On the Subspaces

We saw that Q is nilpotent (which means there is some integer $n > 0$ such that $Q^n \equiv 0$), in particular $Q^2 \equiv 0$. Let $\mathcal{H}$ be the full set of states of the theory, including ghosts and longitudinal photons and the like. Let $\mathcal{H}_3$ be the kernal of Q, that is, all states $|\psi_3\rangle$ in $\mathcal{H}$ for which $Q|\psi_3\rangle = 0$. Let $\mathcal{H}_2 = Q\mathcal{H}$, that is, all states which are Q of something. As $Q^2 = 0$, all states in $\mathcal{H}_2$ are killed by Q and thus in $\mathcal{H}_3$, so $\mathcal{H}_2 \subset \mathcal{H}_3$. Let $\mathcal{H}_0 = \mathcal{H}_3/\mathcal{H}_2$ the coset space. Thus a state in $\mathcal{H}_0$ is a state annihilated by Q modulo states which are Q of something. This is the cohomology of Q. It is also the set of physical states.

My objection to what P&S say is that their $\mathcal{H}_1$ is not really a vector space (it doesn't contain 0, for instance).