Physics 613 Lecture 25 May 1, 2014

Other Issues with the Standard Model

We have now presented the standard model, which includes both the strong interactions via color SU(3) and the electroweak interactions via SU(2)×U(1), and is built on the fundamental particles which are the four gauge particles, the complex doublet Higgs scalar, the three generations of quarks, with the left-handed pieces arranged in three doublets and the right-handed in 6 singlets, and three generations of leptons, with the left handed pieces in doublets, with a negatively charged component and a neutrino, and right handed components in singlets for the charged leptons, but not necessarily for the neutrinos. If the right handed neutrino pieces do exist, they are not coupled to anything within the traditional standard model.

The standard model does not discuss gravitational interactions, and indeed never pretended to. The question of how general relativity and quantum mechanics can be made compatible is very unclear and certainly beyond this course. But there are also a few lacunae more directly involved in particle physics. Three of these we might discuss are

- The strong CP problem, θ terms
- Chiral Anomalies
- Neutrino masses and mixing

Neutrino masses and mixing

Perhaps the one giving the most excitement is neutrino mixing, so let's start with that[1].

I made the rash statement Tuesday than we did not need a CKM matrix for the leptons because the different generations of leptons, electron, muon, and tau, were separately conserved. NOT TRUE! We now have conclusive evidence for neutrino mixing — that is, a neutrino created in the sun as an electron neutrino can arrive at Earth as something else, and a muon neutron created by pion decay in the upper atmosphere can be a different kind of neutrino by the time it reaches detectors deep in a mine somewhere.

[1]In book, 24.4.1-2

A little history might be worth telling. The Sun is powered by a chain of nuclear reactions, the most important part taking four protons and converting them into one helium, two positrons, and two electron neutrinos. As this is the primary source of solar energy, and the solar output is well measured, the rate at which electron neutrinos are produced could be calculated. In the late '60's, Ray Davis set out to measure them by placing 100,000 gallons of perchloroethylene in the Homestake mine, and counting the atoms of Argon created by ${}^{37}\mathrm{Cl} + \nu_\mathrm{e} \rightarrow {}^{37}\mathrm{Ar} + \mathrm{e}^-$ by bubbling helium gas through the tank, which grabs the argon, and the individual atoms are counted by their radioactive decays. On the average, he got 0.41 atoms per day, of which 0.08 were background.

Davis' measured rate was far lower than the calculations from solar models predicted. For many years the controversy was over which one was wrong. We now know that in fact both were right, but 2/3 of the neutrinos emitted as electron neutrinos by the sun were no longer electron neutrinos by the time they reached Earth.

There have been many experiments since that have shown *neutrino oscillations*, that one kind of neutrino evolves into a mixture of the different kinds by itself, not through interactions with other particles. We will explain this.

Massive Neutrinos

Neutrino mixing can only happen if the neutrinos have mass, something which was assumed not to be true for a very long time. We know that in the Dirac equation a mass requires the coupling of the right and left handed pieces of the spinor, so if there were only left handed pieces, there could be no mass[2]. The neutrino could be an ordinary Dirac particle, with independent helicity components, or it could be a Majorana particle, described by a two complex component spinor χ with a mass term $m\chi^T i\sigma_2\chi$ + h. c. A majorana particle is its own antiparticle.

If the neutrinos have mass, and unequal masses, then there is a distinction between the three flavors which might not line up with the distinction provided by the weak isospin raising operator acting on the e^-, μ^- and τ^- respectively. We will call these neutrino states $|\nu_\alpha\rangle$ and there will be an

[2]A Lorentz transformation to an observer moving in the same direction as the neutrino, but faster than it, would see the direction of its momentum reversed but its spin not, so as to reverse the heliicity.

analog of the CKM matrix of the quarks, $|\nu_\alpha\rangle = U^*_{\alpha j}|\nu_j\rangle$, where Greek indices take on the values e, μ and τ and describes weak-isospin flavors, while latin indices are used for $|\nu_j\rangle$ which are the three eigenstates of the mass matrix. The matrix U is known as the PMNS[3] matrix. A particle traveling in empty space has momentum and energy connected by $E^2 - \vec{p}^2 = m^2$, so if we create a neutrino through a weak interaction at $x = 0$ with energy E, of flavor α, and if it travels in the x direction and we detect it at x, the state will be $\psi(t) = \sum_j U^*_{\alpha j} e^{-iEt+ip_j x}|\nu_j, p_j\rangle$ where $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$, assuming the masses are much less than the energy. If this particle is detected by a weak interaction of type β, the amplitude is $\langle\nu_\beta|\psi\rangle = \sum_{jk}\langle\nu_k|U_{\beta k}U^*_{\alpha j}e^{-iE(t-x)-im_j^2 x/2E}|\nu_j, p_j\rangle = \sum_j U_{\beta j}U^*_{\alpha j}e^{-iE(t-x)-im_j^2 x/2Ep_j}$, and the probability is then

$$P(\alpha \to \beta) \propto \left|\sum_j U_{\beta j}U^*_{\alpha j}e^{-im_j^2 x/2E}\right|^2.$$

The x dependence comes from the interference from different m_j's, so that the phases of the different contribution to the sum inside the $||$ differ. If we only had two flavors this would give a function of x having oscillations with wavelength $4\pi E/(m_j^2 - m_k^2)$. Things are a bit more complicated with three flavors, but in many cases two flavors dominate, and in any case you can see the basic idea — oscillations in the probability of conversion from one flavor to another measure the difference in m^2 of the mass flavors. The degree to which the mass flavors line up with the weak isospin flavors, which depend on the PMNS matrix, is not known. It is not even known if the closest matchup has e, μ and τ matched with the lightest, middle, and heaviest mass eigenvectors.

If the neutrinos are dirac particles, the same considerations that we described for the CKM matrix apply to the PMNS matrix, where the nine real variables of the unitary matrix can be manipulated by choice of phase for $|\nu_\alpha\rangle$ and $|\nu_j\rangle$, which eliminates five variables, but that still leaves room for one inherently complex variable. Note also that the phase of oscillations depends on the phase of $U_{\beta j}U^\dagger_{j\alpha}$ so is reversed for $P(\beta \to \alpha)$. This means complexity can give rise to PC or T violation.

If the neutrinos are majorana particles, things are different, because the mass term is not invariant under changes in the phase of χ. So there are only

[3]Pontecorvo, Maki, Nakagawa, and Sakata.

the three phases of the $|\nu_\alpha\rangle$ to fiddle with, and the PMNS matrix has six real variables, three of which must be phases, leading to more ways CP could be broken. This lack of invariance under equal phase transformation of all the leptons also means that the corresponding conserved quantum number, total lepton number, will be violated. Then it is possible to have neutrinoless double beta decay, ${}^A_Z X \to {}_{Z+2}^{\;\;A}Y + e^- + e^-$. Search for this decay of ^{76}Ge had part of the collaboration saying the half-life is $> 1.9 \times 10^{25}$ years, but the other part saying it is 2.3×10^{25} years and not infinite. This would correspond to $m = 0.4$ eV, but is still very controversial.

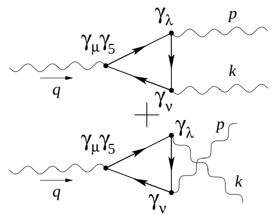
θ Terms, Chiral Anomalies

See Aitchison and Hey, 14.2.4, 18.4

When we discussed QED as an extremely successful perturbative field theory, we mentioned that all the divergences could be canceled by counterterms of the same form as the terms which entered the lagrangian, including field strength renormalization, mass terms, and coupling constant renormalization. This could be understood by looking at the dimensions of operators that preserved the symmetries, including proper lorentz transformations, parity, and Ward identities. But with the weak interactions we have done away with parity invariance, and there is another term which has dimension 4 that does not appear, namely $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, or for QCD, $\theta g_s^2\epsilon_{\mu\nu\rho\sigma}F_a^{\mu\nu}F_a^{\rho\sigma}/64\pi^2$. For QED this is $8\vec{E}\cdot\vec{B}$, which clearly is odd under parity and also under time-reversal (which leaves $\vec{E}$ unchanged but reverses[4] $\vec{B}$, while it is invariant under charge conjugation. Such a term will not arise perturbatively in QED because parity is conserved in the Feynman rules, but also would give no effect perturbatively, because it is a total derivative, $\partial_\mu(4\epsilon_{\mu\nu\rho\sigma}A^\nu\partial^\rho A^\sigma)$, so that the variation of it, integrated over spacetime, is a surface term unaffected by localized δA_ν.

[4]Just think of a classical field due to current flow.

This kind of term arises even in QED if we calculate the divergence of the axial current, which ought to be conserved if all the fermions are massless. If we calculate the triangle diagram with one axial current and two vector currents,



$$\begin{aligned}\mathcal{M}_{\mu\nu\lambda} &= e^2 \int \frac{d^4\ell}{(2\pi)^4} \\ &\quad \mathrm{Tr}\left[\gamma^\mu \gamma_5 \frac{\not\ell - \not k}{(\ell-k)^2}\gamma^\lambda \frac{\not\ell}{\ell^2}\gamma^\nu \frac{\not\ell + \not p}{(\ell+p)^2}\right] \\ &\quad +((p,\nu)\leftrightarrow(k,\lambda)).\end{aligned}$$

If we take the divergence, $\gamma_\mu\gamma_5 \to \not q\gamma_5 = (\not\ell + \not p)\gamma_5 + \gamma_5(\not\ell - \not k)$ which cancels one of the propagators, and then, with a shift of the ℓ integral, $\ell \to \ell + k$, the first term exactly cancels the $((p,\nu)\leftrightarrow(k,\lambda))$. The only problem is, this is a divergent integral $\int d^4\ell/\ell^3$, and the shift of integration variable is not justified. The argument for treating this correctly is rather involved, but the upshot is that

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}.$$

Now if we consider the triangle diagram in electroweak theory, where we need to have the axial current conserved, the contribution from all the fermions must cancel. If the three gauge particles couple with t^b to the gauge b vector or axial vector particle, the contribution is proportional to $\mathrm{Tr}\left[\gamma_5 t^a\{t^b,t^c\}\right]$ summed over all fermions. The γ^5 gives us a -1 for all left handed and $+1$ for all right-handed fermions, so only gauge bosons which distinguish can give trouble. We treat the electroweak gauge bosons in their unbroken state — as the problem comes from divergences, the low energy breakdown is irrelevant. For three SU(2) A's, because $\{\sigma^b,\sigma^c\} = 2\delta^{bc}$ and $\mathrm{Tr}\,\sigma^a = 0$, there is no problem. For one B^μ and two gluons, we get $\mathrm{Tr}(t^a t^b y) = \frac{1}{2}\delta^{ab}\left(\sum_{q_R} y_q - \sum_{q_L} y_q\right)$. For the u and d quarks this gives $4/3 - 2/3 - (2\times 1/3) = 0$ for each color. For the e and ν_e we have $-2-(2\times -1) = 0$, so all is well. For a B and two W's, only the left handed quarks and leptons enter, each generation gives $\mathrm{Tr}\,\sigma^a\sigma^b y = 2\delta^{ab}\sum_{q_L} y_q = 2\delta^{ab}(2\times -1 + 3\times(2\times\frac{1}{7}3)) = 0$, where the -2 is from the leptons, the 3 is from having three colors and the 2 from the two quarks in the generation. Again we get cancellation, but only because each generation includes two quarks and one lepton pair, and the quarks come in three colors. Other anomalies also cancel, so GSW does not

have an axial anomaly problem.

Strong CP problem

Refs: Peccei-Quinn B^0 $\bar B^0$ mixing, AH 21.2

In QCD there is also a possible θ term

$$\mathcal{L}_\theta = \frac{\theta g_s^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F_a^{\mu\nu}F_a^{\rho\sigma}.$$

While it is a bit less obvious, this is also a total divergence, and hence has no effect on the Feynman rules.

But for QCD there is another manifestation of this expression — it is connected to the idea of an instanton. An instanton is a configuration of the gauge fields which corresponds to a pure gauge at infinity in all directions, and is a solution to the Euclideanized equations of motion, but is nonetheless not a pure gauge at finite values of x. As a pure gauge is physically nothing, this corresponds to a transition from vacuum to vacuum with real non-vacuum in between. When one quantizes by feynman path integrals, summing over all possible field configurations, these instantons, which have finite action, can contribute to the amplitudes as if the lagrangian had a theta term. There are such instantons for QCD, in which the gauge lines up in SU(3) space at infinity according to the direction in spacetime. These give rise to the Strong CP problem, resolution in terms of axions, and many more ideas.

Other Things We Didn't Discuss

There are lots of interesting ideas in particle physics that go beyond the standard model, that we didn't have any time at all for. Among them

- Supersymmetry
- Grand Unified Theories (GUTs)
- SeeSaw mechanisms
- Inclusion of Gravity
- String Theory

But, alas, we can't get into this here.

I hope you will continue your interest in particle physics, and there is plenty to learn and plenty to do in this field.