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Bjorken Scaling, Partons

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As we mentioned in considering $e^+e^- \to$ hadrons, hadronic particles are not elementary particles with simple fields interacting with simple $\mathcal{H}_I$ terms. At low energies the interactions are strong and not subject to perturbation theory calculations. At short distances the interactions are weak. If a virtual photon of large q^2 comes along and interacts with part of a hadron, that part may not interact immediately with the rest of the hadron, though as it tries to escape (a longer distance phenomenon) it will need to.

Picture the virtual photon as interacting elastically with a piece of the hadron, which we will call a parton. Then that piece, which has gained a lot of momentum relative to the rest of the hadron, will have long-distance interactions with it which will result in the hadronic matter breaking into pieces. If the initial interaction can be treated independently of the rest, the inclusive cross section will be given by that for elastic proton-parton scattering. If the parton is a fermion, it will be given in terms of the form factors for that parton, but with the initial parton momentum only a fraction, xp^μ, of the proton's momentum. Of course if we take that literally, that implies the mass is xM, which probably doesn't make much sense, but at high momentum transfers maybe any fixed mass will do. The partons final momentum is $xp^\mu + q^\mu$, so again assuming its mass is fixed, $(xp^\mu + q^\mu)^2 = x^2M^2 \Longrightarrow q^2 = -2xp\cdot q = -2xM\nu$, or

$$x = Q^2/2M\nu.$$

Thus the form factors $W_i(Q^2,\nu)$ contain a $\delta(\nu - Q^2/2Mx)$

Now the proton consists of many partons, and each may carry a varying fractions x_j of the proton's momentum, and if we assume they are elementary fermions of charges e_j, the total cross section from all the gluons will be an integral over the parton distribution $f_j(x)$, which gives the likelihood of finding a parton of type j with a fraction x of the proton's momentum. Each parton will give a contribution $e_j^2\delta(\nu - Q^2/2Mx)$ to W_2 if it's there, and $(\nu^2/Q^2)e_j^2\delta(\nu - Q^2/2Mx)$ to W_1. Then

$$W_2(\nu,Q^2) = \sum_j \int_0^1 dx\, f_j(x)\, e_j^2\delta(\nu - Q^2/2Mx) = \frac{1}{\nu}\sum_j x f_j(x) e_j^2\Big|_{x=\frac{Q^2}{2M\nu}} =: \frac{1}{\nu}F_2(x)$$

$$\begin{aligned} W_1(\nu,Q^2) &= \sum_j \int_0^1 f_j(x) e_j^2 \frac{Q^2}{4M^2x^2}\delta(\nu - Q^2/2Mx) \\ &= \frac{1}{4Mx^2\nu}\sum_j x f_j(x) e_j^2\Big|_{x=\frac{Q^2}{2M\nu}} = \frac{1}{2M}\sum_j e_j^2 f_j(x) =: \frac{1}{M}F_1(x) \end{aligned}$$

Thus

$$F_2(x) = x\sum_j e_j^2 f_j(x) = 2xF_1(x),$$

where the relation $F_2(x) = 2xF_1(x)$ is called the Callen-Gross relation.

In the book there is a discussion of how to view this cross section as a sum of a transverse polarization piece and a longitudinal polarization piece. For a real photon we know there are only the transverse polarizations, but we are treating the electromagnetic field A^μ as having four possible polarizations.

Consider a virtual photon with spacelike momentum $q^\mu = (q^0, 0, 0, q^3)$, with $Q^2 = -q^2 = q_3^2 - q_0^2$. The transverse polarizations can be written in terms of the two helicity ± 1 states $\epsilon^\mu(\lambda = \pm 1) = \mp(0, 1, \pm i, 0)/\sqrt{2}$. Define a longitudinal polarization[1] $\epsilon_L^\mu = (q^3, 0, 0, q^0)/\sqrt{Q^2}$, and also a gauge piece[2] $\epsilon_g^\mu = (q^0, 0, 0, q^3)/\sqrt{Q^2} = q^\mu/\sqrt{Q^2}$. So we have four polarization vectors $m = +1, -1, L$ and g,

$$\begin{aligned} \epsilon_{\pm 1}^\mu &= \mp(0,1,\pm i,0)/\sqrt{2} & & (\epsilon_m^*)^\mu(\epsilon_n)_\mu = \zeta_m\delta_{mn}, \quad (1) \\ \epsilon_L^\mu &= (q^3,0,0,q^0)/\sqrt{Q^2} \quad \text{with} & & \sum_m \zeta_m(\epsilon_m^*)^\mu(\epsilon_n)^\nu = g^{\mu\nu}, \quad (2) \\ \epsilon_g^\mu &= (q^0,0,0,q^3)/\sqrt{Q^2} & & \text{with } \zeta_L = 1,\ \zeta_g = \zeta_{\pm 1} = -1. \end{aligned}$$

Now when we calculate $\mathcal{M}$ for e^-p scattering, we have the photon propagator $-g^{\mu\rho}/q^2$ connecting J_μ^e and J_ρ^X. If we wish to ask the contribution of the different polarizations of the virtual photon, we could write this as $-\sum_m \zeta_m \epsilon_m^{*\mu}\epsilon_m^\rho J_\mu^e J_\rho^X/q^2$. This would break the amplitude up into the contribution of each "helicity", but why is the square a simple sum? That is, why is the cross section a sum of transverse and longitudinal pieces, rather than the amplitude is, and the cross section could have interference terms?

One way to understand[3] why there should be no interference is to go to the frame which is the center of mass of the outgoing and incoming electrons

[1] Aitchison and Hey call this $\epsilon^\mu(\lambda = 0)$, but this is a bit confusing as 9.39 and 9.42 differ in sign, and ϵ_L enters the completeness condition (2) below with a negative sign.

[2] Just a name I made up, not generally used.

[3] I need to thank Prof. Schnetzer for this argument.

(even though they don't exist at the same time, they still constitute two timelike forward-directed particles so they have a center of mass frame). Then rotate so that $\vec{k}'$ is in the z direction, and as $\vec{k} = -\vec{k}'$, $q = k' - k = (0, 0, 0, \sqrt{Q^2})$, so everything is along the z axis. If the initial and final helicities are the same, the spin along the z axis has flipped, and the photon must have helicity ± 1. If the spin has not flipped, the photon has helicity zero, and is pure longitudinal. As the electron states are not virtual but have definite, even if unmeasured, spins, there is no interference between longitudinal and transverse virtual photons.

[I must admit that I tried to work out this argument algebraically and failed, seeming to find there is no longitudinal contribution to the electron vertex in the limit $m_e \to 0$ in which we are working.]

Read the discussion in the book, sections 9.1 and 9.2, on Bjorken scaling, which argues that the observation that the transverse photon contribution dominates the longitudinal one is evidence that the partons are spin 1/2 particles. This encourages us in our view that the charged ingredients of hadronic matter are Dirac particles, namely quarks.

Partons as Quarks and Gluons

We saw that the deep inelastic scattering cross sections are determined by the structure functions $F_1(x)$ and $F_2(x)$, with $F_2(x) = \sum_j e_j^2 \, x f_j(x)$, where $f_j(x)$ is the probability of finding a quark of type j with momentum xp^μ, where p^μ is the momentum of the proton. This is really supposed to be thought of in a frame in which the proton is moving near the speed of light, the infinite momentum frame, in which we can imagine that the interactions between quarks and gluons is slowed down. Remember that in the proton's rest frame the partons are strongly interacting and one cannot picture it as three quarks moving slowly. For example, the proton mass is nothing like the sum of twice the u mass and the d mass, each of which is thought to be only a few MeV. In addition to these three quarks, there can be pairs of virtual quarks and antiquarks, and there are gluons, colored but chargeless vector particles which form the foundation of QCD.

So each hadron will have distribution functions for each type of quark and antiquark. Restricting ourselves to the lightest three flavors, for which flavor symmetry $SU(3)_{\text{fl}}$ makes some approximate sense, we would have distributions for a proton of $u(x)$, $d(x)$, $s(x)$ $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$. The proton flavor tells us the total nubmers must satisfy $\int_0^1 (u(x) - \bar{u}(x) = 2$ $\int_0^1 (d(x) - \bar{d}(x) = 1$ and

$\int_0^1 (s(x) - \bar{s}(x) = 0$, By isospin invariance the partition functions for u and d should be interchanged for the neutron, and similarly for $\bar{u}$ and $\bar{d}$, with the strange and antistrange distributions unchanged. So the distribution functions for e^-p and e^-n deep inelastic scattering are

$$\begin{aligned} F_2^{\text{ep}}(x) &= \frac{x}{9}\left(4u(x) + 4\bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{d}(x)\right) \\ F_2^{\text{en}}(x) &= \frac{x}{9}\left(u(x) + \bar{u}(x) + 4d(x) + 4\bar{d}(x) + s(x) + \bar{d}(x)\right) \end{aligned}$$

We might also ask about the total momentum, as we must have $\sum_j \int_0^1 x f_j(x) = 1$, but here we have not only the quarks but the gluons, and gluons apparently carry about 56% of the momentum, but are not contributing to electron scattering via photon exchange because they are uncharged.

Drell-Yan

The Drell-Yan process occurs in proton-proton scattering, in which a $\mu^+ \mu^-$ pair is produced along with hadronic matter which is summed over. As the $\mu^+ \mu^-$ is coupled by means of one photon exchange, and as we can imaging the other end of the photon propagator is coupled to a quark parton from one of the protons and an antiquark parton from the other, we can view this as an integral over distribution functions of $q\bar{q} \to \mu^+ \mu^-$ scattering, which has a total cross section $\sigma(q_a \bar{q}_a \to \mu^+ \mu^-) = \dfrac{4\pi\alpha^2}{3q^2} e_a^2$, where in the proton-proton center of mass frame, the partons have $p_q = x_q(P, 0, 0, P)$, $p_{\bar{q}} = x_{\bar{q}}(P, 0, 0, -P)$, so $q^0 = (x_q + x_{\bar{q}})P$, $q^3 = (x_q - x_{\bar{q}})P$, and therefore $q^2 = 4x_q x_{\bar{q}} P^2 = x_q x_{\bar{q}} s = \tau s$. We need to fold this with the distribution functions, but because the quark and antiquark need to have the same color, and our distribution functions so far have been summed over color, we must divide by 3. Then

$$d^2\sigma = \frac{4\pi\alpha^2}{9\, s\, x_1 x_2} \sum_j e_j^2 [q_j(x_1)\bar{q}_j(x_2) + q_j(x_2)\bar{q}_j(x_1)] dx_1 dx_2.$$

This brings us to the end of chapter 9. Next time, we will return to ideal quantum field theory issues, treating field theory beyond the tree approximation.