Physics 613 Introduction Jan. 21, 2014

1 Introduction

Welcome to Physics 613: Particles

This course is supposed to be an "Introduction to the concepts and techniques underlying current research in elementary particles," which these days means, primarily, an understanding of the Standard Model and possible extensions to it. That, of course, is based on understanding quantum field theory, in particular gauge fields, but none of our three semester sequence on QFT is pre- or co-requesite to this course, though I gather most of you have already had at least one semester in that sequence.

This course will be less formal that the field theory sequence, and will deal with specific phenomena of observed physics, mostly the standard model understanding of particles and their interactions.

Although one might argue that all our understanding of particle physics from before the standard model is now obsolete, I am going to begin with some of that history, both because some of this understanding is still useful heuristics, and because understanding some of these concepts from the older perspective is more transparent. An example will be meeting SU(3) in terms of flavor even though that is now not very important, but SU(3) color is a fundamental ingredient of the standard model. So let me give a bit of prehistory of how we understand the physical world.

1.1 Prehistory

In ancient Greece and India, about 2500 years ago, the idea arose that the material objects we perceive are at a deeper level composed of indivisible atoms. Reductionism is the claim that the behavior of objects is to be understood in terms of the interactions of the components of which they consist, so if materials are made of atomic components, understanding how atoms interact is the way to understand materials. But there was no evidence for millennia that the apparently continuous material of our everyday world really consisted of atoms. It was clear that there were many chemicals, and that these chemicals were mixtures of a smaller set of elements. Early in the 19th Century the list of elements started to resemble what we know now. Dalton's observations that many chemicals consisted of discrete proportions of these elements, rather than arbitrary ratios, gave support to the atomic idea. So by the mid 19th century it was clear that material was made up from a small set (about 100) atoms.

In the last decade of the 19th century, the discovery of radioactivity, the electron, and alpha, beta and gamma decays meant these atoms had an internal structure to be understood. In 1911 Rutherford found that the nuclei of atoms were compact objects about which electrons revolved, and in 1932 Ivanenko found the nucleus to consist of protons and neutrons, which together are called nucleons. So at that point one might have said the physical world consisted of electrons, protons and neutrons, held together by certain forces.

Through most of the 20th century these four forces were described as the gravitational force, the electromagnetic force, the strong force, and the weak force.

gravitational force: The gravitational force holds planets, stars, solar systems and galaxies together, and was the first to be understood, at least non-relativistically, by Newton, thereby re-establishing physics as a mathematical science. Of course our understanding of the force was revolutionized by relativity in the early 20th century, but appears at a classical level to be fully understood, at least by a few people.

But gravity is so weak on an elementary particle scale as to be completely irrelevant to current particle experiments, and is not really included in the standard model. And, in fact, we still have no real understanding of how to treat gravity quantum-mechanically, as is necessary if we were to try to understand its effects in particle physics.

electromagnetic force The electromagnetic force was understood classically in the 19th century as described by Maxwell's equations and interacting with charged matter by the Lorentz force. This is a complete understanding at the classical level, even relativistically, and indeed is even the correct description quantum-mechanically down to subatomic, even subnuclear sizes. In fact we have now found modifications are needed at distances of several attometers (10^{-18} m).

Electromagnetism is responsible for all sub-astronomical physics at scales larger than the nucleus.

strong force The previous two forces were classically understood, but do not explain what keeps the protons and neutrons together in the nucleus. This new interaction is called the strong force, and for a long time had no theoretical understanding. We now think it is described by a non-Abelian gauge theory called quantum chromodynamics, but it is still difficult to calculate its effects except in particular kinematic regions.

weak force The heuristics of the strong force conserved a number of quantities, but these were not in fact conserved in nature. Radioactive $\beta^{\pm}$ decay was assigned to a fourth force, the *weak interaction*.

Even at this level of understanding, we see that the simple idea that everything is made of three kinds of particles is somewhat incomplete, and that the separation between material components and the forces is oversimplified. For electromagnetism is not just a Coulomb potential that binds electrons to nuclei, but also includes a particle of its own, the photon. Nonetheless, it was natural to feel that understanding should proceed by understanding the properies and interactions of electrons and nucleons. But as physicists began looking at energetic particles from cosmic rays and also at modelling the properties of the strong interaction, they learned that there were other, unstable particles. Muons were prominent is cosmic rays, and pi mesons were created in accelerators and also explained the large distance behavior of the nuclear force.

1.2 The view of 50 years ago

So if we ask how fundamental physics appeared 50 years ago, we found four very different fields. The few people interested in general relativity were astronomers, and had virtually no interaction with particle physicists. As the electrons did not participate in the strong or weak interactions, and as those interactions are short range, if one confined one's attention to electrons and possibly to low energy processes involving nuclei as charged particles, it was sufficient to study quantum electromagnetism. The nonrelativistic treatment of atomic physics via Schrödinger's equation and the Coulomb potential could be partially relativized by the Dirac equation for the electron, but in fact one could go much further by treating the electron as well as the photon in terms of quantum field theory. Taking Maxwell's theory and Dirac's hamiltonian as involving quantized fields gives a solvable framework

for noninteracting electrons and photons, and adding a term describing the Lorentz force gives a complete theory, though it is no longer exactly solvable. Still, the interaction, which is proportional to the electron charge $-e$, can be treated perturbatively, as the relevant expansion parameter, $\alpha := e^2/4\pi\hbar c \approx 1/137$ is small. As we shall see, it is possible to calculate relevant physical processes order by order in α, and for many purposes just the lowest order perturbation is sufficient. Most impressively, heroic calculations over the last 65 years have calculated the magnetic moment of the electron to 5th order in perturbation theory, yielding the correct experimental value to one part in a trillion! This extremely successful theory is called Quantum Electrodynamics (QED).

The weak interactions, which are responsible for beta decay, such as

$$^{14}\mathrm{C} \longrightarrow {}^{14}\mathrm{N} + \mathrm{e}^- + \bar{\nu},$$

of carbon 14 into nitrogen, an electron, and a neutrino, does not come from a physical theory which is known classically. If, however, one thinks of the process as one neutron decaying to a proton, electron and neutrino, which is in fact how a neutron decays, and assigns fields to each of these particles, it is not hard to postulate an interaction which correctly describes the observed data, using only the first order perturbation in the very weak strength of the interaction. So weak interaction phenomenology was quite successful by 1960. But a problem remained: the quantum field theory which worked so well to first order was inconsistent at higher orders in perturbation theory, more precisely, it was a non-renormalizable theory.

The situation was even worse for the strong interactions. One wished to explain the properties of nuclei, but also, and perhaps more accessible, one could examine scattering data such as proton-proton scattering amplitudes, and also proton-neutron, and by means of deuterium, even neutron-neutron. From this scattering data at low energies, one can extract a nonrelativistic potential, and two features called out for inclusion in any understanding of the strong interactions. One was that at distances comparable to a fermi (femtometer), the potential could be explained by the exchange of a particle of mass of roughly 100 MeV/c^2, which we now understand to be a spin 0 particle, π, which, though unstable with a lifetime of about 26 nanoseconds, could be incorporated into a field theory similar to QED. But unfortunately the coupling constant analogous to α is $g^2_{\pi NN}/4\pi \approx 14$ instead of $\alpha \approx 0.007$. Thus there was virtually no chance that perturbation theory would work.

The second observation that emerged from the data is that the strong forces between nucleons depended on which nucleons were involved in a well defined way. So for example, a two nucleon system in a given state should have the same energy (ignoring the Coulomb energy) regardless of whether we have two protons, two neutrons, or one of each, although in the last case, states where the two nucleons are in the same state are possible, whereas for two identical particles Fermi forbids that state.

This is an indication that there is a symmetry of the strong interactions under which a proton state is rotated into a mixture of a proton and a neutron, just as under rotations a spin up electron at rest is rotated into a mixture of spin up and spin down. This symmetry is known as isospin.

Having a symmetry of the dynamics is very useful, especially when your dynamical theory is such that you can't actually calculate anything from the theory, as seemed to be the case for the strong interactions. If the theory has a symmetry, we may expect[1] that the states form representations of the symmetry group, just as atomic states are classified in represenions (labelled by the total angular momentum) of the rotation group. During the '50's and '60's, while experimentalists were overloading us with new hadronic[2] resonance states, the most useful thing we could do with such information was to look for symmetries.

1.3 Symmetries

So consideration of the symmetries of the fundamental dynamical laws is a very important part of understanding particle physics. A symmetry transformation is a set of changes on the basic fields of the theory, but also a transformation matching states before and after the symmetry. For example, if we were to consider a symmetry of interchanging neutrons and protons (which is a more limited symmetry than the isotopic spin symmetry we mentioned above), it would define new proton and neutron fields

$$\begin{aligned} \psi'_p(\vec{x},t) &= \psi_n(\vec{x},t) \\ \psi'_n(\vec{x},t) &= \psi_p(\vec{x},t) \end{aligned}$$

[1]If we don't have spontaneous symmetry breaking. This will be discussed much later.

[2]Hadrons are particles affected by the strong interactions, including nucleons, pions, and many more to come, but excluding the electron, its neutrino and other leptons, and the photon, and their various cousins.

but it would also replace a state with 3 protons and one neutron with a state of 3 neutrons and one proton. This does not mean every state has this symmetry (*i.e.* is unchanged by the symmetry transformation) but rather that the transformed state evolves as the transform of the evolution of the original state. That is, it obeys the same physical laws, and in particular has the same mass. This is an example of a *discrete global* symmetry. Rather than discrete, a symmetry can be *continuous*, as for example rotations through an arbitrary angle. Global means that the same transformation acts on the fields at each space-time point, and on the states of the system, in the same way. The alternative, local symmetry, is not so apparent, as it does not act simply on the states of the system, and acts independently on the fields at each space-time point. We shall see that the gauge invariance of Maxwell theory is such a symmetry, and that such symmetries are the basis of the non-Abelian gauge theories which we will see are now the foundation of the Standard Model.

As a symmetry is a transformation that leaves the dynamical equations unchanged, or maps all possible states into other possible states, the set of symmetry transformations satisfies the mathematical postulates for a group with multiplication defined by composition. Clearly if a mapping leaves the physics unchanged, so does the inverse mapping, and if two mappings do so, so does the composition, that is, first doing one mapping and then the other. Of course doing nothing is the identity of the group.

In addition to true symmetries, we may have approximate symmetries. That is, we may consider mappings of the fields and/or states which make only small changes to the full dynamics, changes which might usefully be ignored in some contexts. For example, the symmetry under isospin mentioned above is a symmetry at best only of the strong interaction, as clearly protons and neutrons behave quite differently for their Coulomb interaction. It is only because the electromagnetic interactions between two nucleons is much weaker, at small distances, than the strong interaction that isospin is worth discussing. We will see that there are several such approximate symmetries which are useful in particle physics.

1.4 Symmetries of Particle Physics

1.4.1 Poincaré Invariance

So let us begin by discussing which symmetries (and approximate symmetries) are useful to discuss in high energy physics.

First, of course, we will assume that the fundamental laws of physics are the same in all directions. This does not mean that you can conclude that, just because if you throw a ball straight up it will come back and hit you, then if you throw a ball forwards it will come back to you. When we rotate the physics, we need to rotate the whole situation, including the position of the Earth. In particle physics we usually assume the background situation is the vacuum, and we assume it is unchanged under rotations. Some of the ancient Greeks did not respect this abstraction.

Equally assumable is the homogeneity of space, the laws are unchanged by translation — an experiment done in New Jersey gives the same result as in Geneva (assuming gravity and other environmental issues don't affect the results).

In addition, we assume the homogeneity of time, that two experiments performed at different times will give the same results. We now know this is not cosmologically correct, that the universe was vastly different than now 13 billion years ago. But in discussing particle physics, we assume we can ignore this, as well as other features of general relativity. Invariance under rotations, spatial translations and time translation has been understood for a very long time.

Galileo taught us that the symmetry of physics is more than that, that physics is invariant under uniform relative motion. That is, two observers, one of whom is moving with constant velocity with respect to the other, and not rotating, have the same physical laws. Thus the group of symmetry transformations acting on spacetime is

$$\begin{aligned} x'_j &= \sum_k R_{jk} x_k + v_j t + C_j, \\ t' &= t + C \end{aligned}$$

where R is a rotation matrix, an orthogonal matrix with determinant 1, $\vec{v}$ is a constant relative velocity, $\vec{C}$ is the displacement of the origins of the spacial coordinates at $t = 0$, and C a difference in the setting of the clocks, but not the rate at which they tick. This symmetry group, which has 10 parameters, is called Galilean relativity.

Einstein, of course, told us that we had the same set of symmetries, but Galileo's transformation laws were not quite correct. In the Special Theory of Relativity, we treat time as a fourth coordinate, or rather the zero'th, setting $x_0 = ct$, with c the velocity of light, which we can set equal to 1 by choosing to measure time in units of meters, with one second equal to 299,792,458 m. We will use relativistic notation, where what had been called x, y, and z are now called x^1, x^2 and x^3, and $x^0 = x_0$. The four components x^μ, with $\mu = 0, 1, 2, 3$, together form a contravariant vector.

Because the fundamental requirement of special relativistic transformations is that they map x^μ into x'^μ leaving $(\Delta x^0)^2 - \sum_{j=1}^3 (\Delta x^j)^2$ invariant (that is, $(\Delta x^0)^2 - \sum_{j=1}^3 (\Delta x^j)^2 = (\Delta x'^0)^2 - \sum_{j=1}^3 (\Delta x'^j)^2$), it is convenient to define a constant metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

where the rows and columns are labelled 0 to 3. That is, $g_{00} = 1$, $g_{11} = g_{22} = g_{33} = -1$, and all other components are 0. For each contravariant vector V^μ we define a covariant vector V_μ with $V_0 = V^0$, and $V_j = -V^j$ for $j = 1, 2, 3$. Then $V_\mu = \sum_\nu g_{\mu\nu} V^\nu$.

Having Lorentz symmetry in our theory will mean that the basic laws will need to be covariant under Lorentz transformations, and having translational symmetry will mean there should appear no explicit dependence on the spacetime coordinates.

We learned from Noether's theorem that every continuous symmetry generates a conserved current j^μ (with $\sum +\mu \partial_\mu j^\mu = 0$) and a conserved quantity $Q = \int d^3x j^0(\vec{x})$. For the translations, these are the total momentum $\vec{P}$ and energy, which is the zero'th component, P^0. The Minkowski square is thus $\sum_\mu P_\mu P^\mu = E^2 - \vec{P}^{\,2} = m^2$, where m is the rest mass of the whole system, but for a single particle, it is the conserved mass of the particle. The conserved current is the Energy-Momentum (or stress-energy) tensor $T^{\mu\nu}$. The conserved Noether quantity corresponding to rotations is the angular momentum, which as quantum mechanical operators are the space-space part of the generators of the Lorentz group. So a single particle state will have a conserved angular momentum, and if it is a massive particle in its rest frame, that will be its spin, quantized as we learned in quantum mechanics. So particles will have a definite half-integral spin.

1.4.2 Electric charge

Another quantity which we strongly believe to be conserved is electric charge. This is related to the local symmetry of gauge invariance of the electromagnetic field $A_\mu \to A_\mu + \partial_\mu \Lambda$, which we will discuss later.

1.4.3 Isospin

For the strong interactions, we have suggested that isospin symmetry says that physics should be invariant under a transformation that treats the proton and neutron as two components of an isospinor, transforming under rotations in isotopic spin space in the same way as an ordinary spinor transforms under rotations, i.e. under SU(2) transformations. As for angular momentum, the commuting operators which we can diagonalize are $(\vec{I})^2$ and I_z, representations having $(\vec{I})^2 = I(I+1)$ and I a half integer, with $-I \le I_z \le I$, and with $2I+1$ particles in the multiplet. Thus the nucleons are an $I = \frac{1}{2}$ doublet. But the ground state nuclei ${}^{14}_{6}$C and ${}^{14}_{8}$O form an $I = 1$ multiplet together with the first excited state of ${}^{14}_{7}$N, while the ground state of the nitrogen nucleus is an isospin 0 state.

In the '50's particle accelerators had enough energy to create particles that were not previously known, in addition to the pion, which had been found in cosmic rays in 1947. These particles are not stable, and indeed some were so short-lived as to be seen only as resonant states in particle scattering. The pions come in three charges, π^+, π^0 and π^-, forming an isotriplet. Scattering pions off nucleons showed a set of baryon[3] resonances called the Δ's, four of them, Δ^{++}, Δ^+, Δ^0 and Δ^-, which forms an isospin 3/2 multiplet at ∼1232 MeV.

Isospin conservation permits us to make predictions even if we know little else about the strong interactions. For example, consider π nucleon scattering. The initial state consists of an $I = 1$ pion and an $I = 1/2$ nucleon, which must be added as we do for angular momentum, giving $I = 1/2$ and $I = 3/2$.

[3]The proton and neutron are the lightest baryons, and as their masses (∼ 940 MeV) are much heavier than the electron's (0.511 MeV), and of the pions (∼ 140 MeV), they are heavy, $\beta\alpha\rho\upsilon\varsigma$ in Greek, while the electron and its relatives are called leptons, after $\lambda\epsilon\pi\tau\omega\varsigma$ = thin in Greek, and the pions and its relatives are called mesons, $\mu\epsilon\sigma o\varsigma$ = in the middle. But baryon number is apparently conserved, with the nucleons having baryon number 1, the pions baryon number 0, and the leptons are distinguished as being fermions not having strong interactions. So the names stuck even though the heaviest lepton (m=1777 MeV) is heavier than the nucleons, and the B meson group are 5–6 times heavier than a proton.

At a center of mass energy near the Δ mass, the scattering cross section is dominated by the resonance, which is an $I = 3/2$ intermediate state. We can compare scattering of each of the three pions with each of the two nucleons, all at the same angle of scattering, or all the total cross section. There are 6 possible initial states (though π^0 is not really available as an incoming beam, as the π^0 decays in 10^{-16} seconds) and 6 possible final states. Charge conservation restricts us to 10 possibilities, 6 with accessible initial states, for measureable cross sections. The amplitude will then be

$$\sum_{I=1/2}^{3/2} \langle \pi_f, N_f | I \rangle \langle I | \pi_i, N_i \rangle$$

and the cross section the absolute square. In π^+ p scattering, $I_3 = 1 + \frac{1}{2} = 3/2$ so the intermediate state is pure isospin 3/2 and fully couples to the Δ^{++} resonance, but for π^- p, we must add isospin 1 and isospin $\frac{1}{2}$, which gives both $I = 3/2$ and $I = 1/2$ intermediate states. To find the overlaps, we use Clebsch-Gordon coefficients $(j_1 J_2 m_1 m_2 |\, j_1 j_2 J M)$. So

$$\begin{aligned}
\sigma(\pi^+ p \to \pi^+ p) &= \left(1, \frac{1}{2}, +1, +\frac{1}{2}\right| 1, \frac{1}{2}, \frac{3}{2}, \left.\frac{3}{2}\right)^2 \sigma_{3/2} = \sigma_{3/2} \\
\sigma(\pi^- p \to \pi^- p) &= \left(1, \frac{1}{2}, -1, +\frac{1}{2}\right| 1, \frac{1}{2}, \frac{3}{2}, \left.-\frac{1}{2}\right)^4 \sigma_{3/2} \\
&\quad + \left(1, \frac{1}{2}, -1, +\frac{1}{2}\right| 1, \frac{1}{2}, \frac{1}{2}, \left.-\frac{1}{2}\right)^4 \sigma_{1/2} \\
&= \frac{1}{9}\sigma_{3/2} + \frac{4}{9}\sigma_{1/2} \approx \frac{1}{9}\sigma_{3/2} \\
\sigma(\pi^- p \to \pi^0 n) &\approx \left(1, \frac{1}{2}, -1, +\frac{1}{2}\right| 1, \frac{1}{2}, \frac{3}{2}, \left.-\frac{1}{2}\right)^2 \\
&\quad \times \left(1, \frac{1}{2}, 0, -\frac{1}{2}\right| 1, \frac{1}{2}, \frac{3}{2}, \left.-\frac{1}{2}\right)^2 \sigma_{3/2} = \frac{1}{3} \cdot \frac{2}{3}\sigma_{3/2} = \frac{2}{9}\sigma_{3/2}
\end{aligned}$$

So we are able to predict two other cross sections from the $\pi^+ p \to \pi^+ p$ one, with

$$\sigma_{\pi^+ p \to \pi^+ p} : \sigma_{\pi^- p \to \pi^- p} : \sigma_{\pi^- p \to \pi^0 n} = 9 : 1 : 2.$$

Pions are not stable particles. The neutral pion decays into two photons, $\pi^0 \longrightarrow \gamma + \gamma$, and as the photons have spin ± 1 along the direction of travel, a pion at rest can have $s_z = 0$ or 2. Measuring the polarization of the photons shows the π^0 has spin 0.

This decay cannot proceed by the strong interaction, because photons don't participate in the strong interaction, but it can go through the electromagnetic interaction. The mean lifetime is 8.3×10^{-17} s, which is too short to measure directly but still long compared to decays (such as the Δ's) which can proceed by the strong interactions.

The π^+ and π^- have lifetimes of 2.60×10^{-8}s, much longer, decaying into muons and neutrinos. Muons ($\mu^\pm$) are particles that behave just like positron and electron, except their masses are 105.7 MeV, rather than 0.511 MeV. They were discovered in cosmic rays in 1937, and they decay in 2.20 microseconds, apparently into an electron or positron and nothing else. But that would violate energy and momentum conservation, so there must be invisible particles in the decay products. This was not new, because in nuclear beta decay, ${}^A_Z X \to {}^{\ A}_{Z+1} Y + e^-$, the electons were observed to come out with a range of energies rather than the fixed energy which momentum and energy conservation would require. So Pauli postulated (in 1930) that there must be light neutral particle as well as the electron, which he called the neutron. But the same name was used by Chadwick when he discovered the particle we now call the neutron, and Fermi renamed Pauli's particle the neutrino, little neutron. But it is not similar to the neutron, as it does not participate in the strong interactions. We will have a lot to say about recent developments in neutrino physics much later in the course. But for now, let's accept that there is an electron neutrino and a muon neutrino, and

$$\pi^+ \to \mu^+ + \nu_\mu, \qquad \pi^- \to \mu^- + \bar{\nu}_\mu$$

where we note that neutrinos each have its antiparticle $\bar{\nu}$. (The π^- is the antiparticle of the π^+, the π^0 is its own antiparticle, as is the photon.) The muon decays

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu, \qquad \mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu.$$

These decays involve the weak interaction, and that is why the lifetimes are so long, on the scale of particle physics, despite plenty of energy being available.

We should pause here to note that the properties of antiparticles are the same as for their particles, except for opposite charges, so the two μ particles have the same mass and the decays above occur at the same rate. This

implies a symmetry transformation, called charge conjugation, which maps each particle into its antiparticle. Charge conjugation is a symmetry of the strong and electromagnetic interactions, but, it turns out, not of the weak interactions, and in fact the identity of the decay rates above (which are due to the weak interaction) are guaranteed by a more complex symmetry.

Note that, if we assign e-lepton number 1 to the electron and ν_e, and μ-lepton number to μ and ν_μ, the decays above conserve both e-lepton number and μ-lepton number. Whether this is more generally true will be dealt with later.

1.4.4 Parity

Before we continue pursuing history further, let us note that the assumption that physics is invariant under Poincaré transformations is a bit ambiguous. If we define the Lorentz group to be the linear transformations of space-time that leave the Minkowski x^2 invariant, it contains not only transformations that one could induce by rotations or by accelerations to a new inertial system, but also *Parity* (P), that is, $\vec{x} \to -\vec{x}$ with t unchanged, and also *time reversal symmetry*, (T), which maps $t \to -t$ leaving the spatial components $\vec{x}$ unchanged. Both parity and time reversal are symmetries of the strong and electromagnetic interactions. That means physical processes shown in a mirror should look like they could have happened observed directly, so in particular states of a single particle at rest should be transformed into possible states of that particle, though the spin might be reversed. For a scalar particle the state must represent the same physical state, but the quantum mechanical state might have its sign changed, as it is the square that matters. The sign is called the *intrinsic parity* of the particle.

What is the parity of the pion? If a π^0 at rest decays into two photons, the parity of the final state involves the direction of the photons $\vec{k}$ and the two polarization vectors ϵ_1 and ϵ_2. The two possible rotation-invariant functions are proportional to $\epsilon_1 \cdot \epsilon_2$ and $(\epsilon_1 \times \epsilon_2) \cdot \vec{k}$, which give opposite distributions in the angle between the planes of polarization. But these functions are scalar and pseudo-scalar respectively, so have parity $+1$ and -1 respectively. The polarizations are hard to measure directly, but the process in which each photon converts to a $e^+ e^-$ pair is measurable and shows the decay is described by the second function, which implies that the π^0 has negative parity and parity is conserved in the decay, and suggests it is conserved by the strong and electromagnetic interactions.

1.4.5 Strangeness

In the same year as the pion was discovered, another set of spin 0 particles with mass about half the proton's was discovered. These decayed quite slowly, around 10^{-10}s, into two or three pions, despite having plenty of energy to do so. Thus there had to be a reason the strong interaction could not do this, and Pais postulated a new quantum number conserved by the strong interactions, called *strangeness*, because none of the previously known particles had any. In particular, there seemed to be two particles, $\theta^+ \to \pi^+ + \pi^0$ and $\tau^+ \to \pi^+\pi^+\pi^-$. These had the same mass and other properties, so appeared to be the same particle, but a scalar decay into two pions must have parity +1, while three negative parity pions can only have positive parity if the wave-function $\psi(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ is a pseudoscalar, so must involve $(\vec{k}_1 \times \vec{k}_2) \cdot \vec{k}_3$. But this is antisymmetric under interchange of any two momenta, while the two π^+'s, being indentical bosons, need to enter symmetrically. So either we have two particles with opposite parities but otherwise the same properties, or parity is not conserved and the same particle is responsible for both observed decays. Of course we now know that parity is not conserved by the weak interactions, and as these decays are relatively long-lived, it is enough to claim parity is not a symmetry of the weak interactions. The τ^+ and the θ^+ are the same particle, now called the K^+. It is part of an isodoublet with the K^0, both with strangeness +1. Their antiparticles K^- and $\bar{K}^0$ have strangeness −1.

In 1950 another interesting particle was observed in cosmic rays, a neutral particle called the Λ^0, mass 1116 MeV, which decayed into a proton and a π^-, but with a lifetime of 2.6×10^{-10}s, much too long for a strong interaction. However, the facts that sufficiently energetic proton proton collisions can result in a final state with $\Lambda^0 + K^+ + p$ suggests that the Λ^0 is a baryon with strangeness −1, so the total strangeness in the final state is zero, as it was in the $p\ p$ initial state. This strangeness assignment is also consistent with a strong production by π^- beams, $\pi^- + p \longrightarrow \Lambda^0 + K^0$, and K^- beams, $K^- + p \longrightarrow \Lambda^0 + n$.

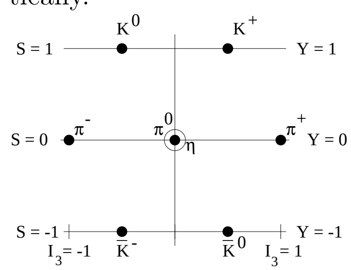
In nuclear reactions, the number of nucleons, A, is conserved. In fact, if we generalize to baryons, we have so far seen no violation of overall baryon number conservation, where we are assigning baryon number $B = 1$ to p, n, Δ, Λ^0, and many other resonances. The pions and other mesons have baryon number 0, as do the leptons.

With the increasing power of accelerators, lots of new particles were pro-

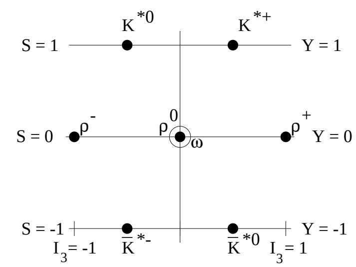
duced. An isotriplet of strangeness −1 baryons, Σ^+, Σ^0 and Σ^- with mass 1193 ± 4 MeV, with spin $\frac{1}{2}$, and an isodoublet of baryons Ξ^0 and Ξ^- with mass ≈ 1320 MeV, both with spin $\frac{1}{2}$. There were also baryons of spin $\frac{3}{2}$, besides the unstrange Δ with isospin $\frac{3}{2}$, an isotriplet Σ^* at 1384, strangeness −1, and an isodoublet Ξ^* at 1533 MeV with strangeness −2.

There were more bosons as well. An odd parity spin 0 (pseudoscalar) isoscalar η with mass 548 MeV, no strangeness, fits in with the π nonstrange isotriplet, the strangeness +1 isodoublet K^+ and K^0, and their antiparticles $\bar{K}^-$ and $\bar{K}^0$ with strangeness −1. There were also spin 1 particles, a nonstrange isotriplet ρ at 770 MeV, a nonstrange isosinglet ω at 782 MeV, and excited isodoublets K^* and $\bar{K}^*$ at 892 MeV.

Let us plot out these particles with I_3 horizontally and strangeness vertically.

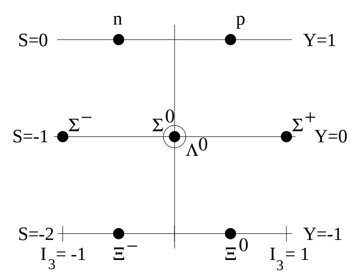


Spin 0, $P = -1$.

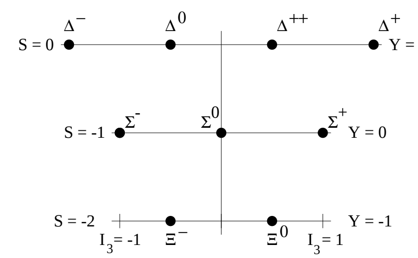


Spin 1, $P = -1$.

The mesons



Spin 1/2, $P = 1$.



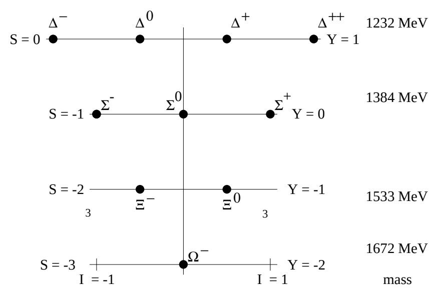
Spin 3/2, $P = -1$.

The baryons

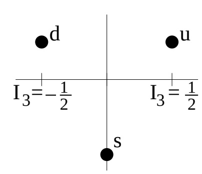
It certainly looks like we have four families of particles, and within each family the different particles are distinguished by the I_3 and strangeness values. Might there be a symmetry group for which these families are multiplets? Notice that for the first three families, there is a 3-fold rotational symmetry of the diagram, and that we must have a group with two commuting generators for I_3 and S, although we see that for the baryons, it might be better to add 1 to S. If we define *hypercharge* $Y = S + B$, the origin of the rotation for the diagrams is $Y = I_3 = 0$. Note also that the charge of the particle is given by $Q = I_3 + Y/2$.

Gell-Mann in 1964 proposed an approximate symmetry of the strong interactions with a symmetry group SU(3), that is, unitary complex matrices with determinant 1, as a generalization of the SU(2) of isospin. Mathematicians had long before discussed representations of SU(3) and other semisimple Lie groups, and the mesons and spin 1/2 baryons fit into the adjoint (defining) representation.

The nine spin 3/2 baryons, however, do not fit any represention of SU(3), although there is one with one additional particle, called the decouplet. Gell-Mann then predicted the existance of a new particle, the Ω^-. This SU(3) symmetry is clearly not as good as SU(2), for the particles have masses differing by ~ 100 Mev rather than a couple of MeV as the isospin partners do, but the three isospin spin 3/2 multiplets were evenly spaced in mass, so Gell-Mann was able to predict, approximately, the mass of the Ω^- near the 1672 MeV it has.



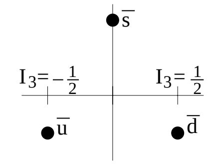
The fundamental representation of SU(3), that is the three dimensional space on which the group elements act as the unitary matrix should, is, of course, 3 dimensional. Isospin is the upper left 2×2 piece of the 3×3 SU(3) transformation matrix, so the top two components, the u and the d, form an isodoublet. This is the quark representation.



The quark representation

The complex conjugate is also a representation of SU(3), but it is not equivalent to the quark representation. This is different from the case of SU(2), where the fundamental representation is the doublet, and is equivalent

to its conjugate. It is the antiquark representation. All finite-dimensional representations can be built from products of the fundamental representation and its conjugate, so one intuitive idea of how to understand the hadronic particles is as collections of quarks, in the fundamental representation, and antiquarks, in the conjugate.



The antiquark representation

Gell-Mann proposed the quarks more as mnemonics than as actual particles, but deep inelastic scattering experiments, the large momentum transfer scattering of electrons off nucleons without regard to the final state of the hadrons, showed that indeed the proton behaved like it had hard, point-like particles within, *partons*, and the quarks were then regarded more as real particles, which could serve as the partons.

The baryon decouplet can be made from three quarks, so the quarks must have baryon number $B = 1/3$. As the Δ^{++} must have three u quarks, the states must be totally symmetric in the quark flavors. The nucleon octet can also be made from three quarks, combined in a mixed symmetry, in the same way that a state of three spin 1/2 objects can be spin 1/2 or spin 3/2 depending on the symmetry of the combination. The mesons can then be made of a quark and an antiquark.

From the representations, we see that the quantum numbers of the quarks are given by

Name	I_3	Y	B	Q	S
u	$+1/2$	$1/3$	$1/3$	$2/3$	0
d	$-1/2$	$1/3$	$1/3$	$-1/3$	0
s	0	$-2/3$	$1/3$	$-1/3$	-1
$\bar{u}$	$-1/2$	$-1/3$	$-1/3$	$-2/3$	0
$\bar{d}$	$+1/2$	$-1/3$	$-1/3$	$+1/3$	0
$\bar{s}$	0	$+2/3$	$-1/3$	$+1/3$	$+1$

If you take one quark with 3 possible flavors and one antiquark with 3 possible flavors, there are 9 mesons one could have, say in an $L = 0$, with spins antialigned for $J = 0$ or with spins aligned for $J = 1$. Of the nine possibilities, one is invariant under SU(3) transformations, namely $u\bar{u} + d\bar{d} + s\bar{s}$, while the other 8 have the quantum numbers we showed for the pseudoscalar and vector octets. So that picture of the mesons as $q\bar{q}$ states works beautifully.

For the baryons, made of three quarks with spin 1/2, the 27 combinations of SU(3) indices can be split into 1 totally antisymmetric, 10 totally symmetric, and 2 sets of 8 combinations of mixed symmetry. A total spin of 1/2 from three spin 1/2 objects requires a state of mixed symmetry, as the totally symmetric combination is spin 3/2, and there is no totally antisymmetric state. The mixed symmetry SU(3) function and the mixed symmetry spin function can be combined into totally symmetric, totally antisymmetric, or mixed symmetry, and as the quarks are fermions, we might expect the nucleon octet is the antisymmetric combination. But when we look at the spin 3/2 decouplet, both the spin combination and the SU(3) combination must be symmetric, so this seems to contradict the fermi statistics for the quarks.

Several ways around this problem were suggested, but the best of these, and indeed a primary basis of the standard model, is to assume that the quarks have another quantum number of which we have been totally unaware. This quantum number is known as color, though of course it has nothing to do with real colors, so when I say a quark can be red, green or blue, this is only an analogy to the primary colors of visible light. So in addition to having three flavors, the quarks come in three colors, and we can consider unitary transformations in color space, color SU(3), which is separate from the flavor SU(3) we have just discussed[4].

For the decouplet particles, to get overall fermi statistics, the wave function of the three quark colors must be totally antisymmetric, and that combination is invariant (transforms as a singlet) under color SU(3), so of course we can say it is white. The observed particles are consistent with all being white, as the baryons can all be totally symmetric in $SU(3)_{\text{flavor}} \times SU(2)_{\text{spin}}$, and the mesons can be combinations of quark-antiquark of the opposite color (*e.g.* blue anti-blue).

The SU(3) of color has turned out to be much more inportant than the approximate flavor symmetry SU(3). In fact, starting in 1974 new flavors of quarks were discovered, having masses wildly different from the three quarks of the late '60's. These are called charmed, top and bottom quarks, though for a while the last two were sometimes called truth and beauty. The SU(6) transformations one could imagine rotating an up quark into a top quark of mass 173 GeV is so far from being a symmetry that it is totally useless.

[4]This might suggest an SU(9) symmetry of the nine states of a quark, but this is not useful.

On the other hand, the SU(3) of color is now considered to be an exact symmetry of the underlying dynamics, a non-Abelian gauge theory which is a generalization of electrodynamics, an Abelian gauge theory. Color SU(3) is not directly observable in the particle spectrum because, due to a feature of the non-Abelian theory called infrared slavery, only colorless combinations of particles can separate from the rest. This is confinement.

Of course the theory of color gauge invariance had to be called *quantum chromodynamics*, or QCD.

The quarks are the SU(3) analogs of the electrons for QED, and the analogs of the single photon are eight different gluons. But as gluons generate the SU(3) transformations they transform in the adjoint or octet representation, not the singlet, and so they are confined within hadrons.

It turns out that non-Abelian gauge theories are what particle physics is all about, and in addition to QCD, the standard model incorporates another non-Abelian gauge symmetry, $SU(2)\times U(1)$, known as Glashow-Salam-Weinberg or electroweak theory. The photon is actually one of the gauge particles, along with the $W^\pm$ and Z^0 vector mesons. The analogs of the charges in QED for this electroweak theory are all the quarks and leptons. In addition to the two families, e and μ which we already discussed, there is another heavy electron called the tau τ with mass 1777 MeV, and its neutrino.

We have now been introduced to all the particles of the standard model except two, one of which is the graviton, but that really plays no role in the standard model as we don't know how to make a theory of quantum gravity. The other is the long awaited but recently discovered Higgs particle, a scalar neutral particle with mass 125 GeV which, as we shall see, is the visible remnant of the field which gives mass to all the other elementary particles.