Physics 613 Spring, 2014
Homework #3. Due Feb. 17 at 4:00

1: Consider the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^\dagger (D^\mu\phi) - m^2\phi^\dagger\phi$$

for a complex scalar field ϕ of charge q, where the covariant derivative $D_\mu\phi :=$ $\partial_\mu\phi + iqA_\mu\phi$, and as usual $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$.

- Derive the classical equations of motion for ϕ.
- Derive the classical equations of motion for A_μ.
- Find the conserved electromagnetic current.
- Verify that the current is conserved if the fields satisfy the equations of motion.

2: In Lecture 6, I claimed that assuming the commutations relations

$$[\phi(\vec{x}), \phi(\vec{x}\,')] = 0, \qquad [\phi(\vec{x}), \pi(\vec{x}\,')] = i\delta^3(\vec{x} - \vec{x}\,'), \qquad [\pi(\vec{x}), \pi(\vec{x}\,')] = 0 \quad (1)$$

for the real scalar field $\phi(\vec{x})$ at equal times implies

$$[a(\vec{k}), a(\vec{k}\,')] = 0, \quad [a(\vec{k}), a^\dagger(\vec{k}\,')] = (2\pi)^3\delta^3(\vec{k} - \vec{k}\,'), \quad [a^\dagger(\vec{k}), a^\dagger(\vec{k}\,')] = 0. \quad (2)$$

for the coefficients $a(\vec{k})$ and $a^\dagger(\vec{k})$. Show that this is true, by inserting the expansions for ϕ and π, taking the double fourier transform which undoes the dual inverse transform of the expansions, and taking the suitable linear combinations of the three equations. It might be helpful to reverse the dummy variable $\vec{k}$ in the $a^\dagger$ pieces.