Vector Identities

These are from the cover of Jackson:

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right) \tag{1}$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{A}\cdot\vec{B}\right)\vec{C} \tag{2}$$

$$\left(\vec{A}\times\vec{B}\right)\cdot\left(\vec{C}\times\vec{D}\right) = \left(\vec{A}\cdot\vec{C}\right)\left(\vec{B}\cdot\vec{D}\right) - \left(\vec{A}\cdot\vec{D}\right)\left(\vec{B}\cdot\vec{C}\right) \tag{3}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\Phi\right) = 0 \tag{4}$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = 0 \tag{5}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \nabla^2 A \tag{6}$$

$$\vec{\nabla}\cdot\left(\Phi\vec{A}\right) = \vec{A}\cdot\vec{\nabla}\Phi + \Phi\vec{\nabla}\cdot\vec{A} \tag{7}$$

$$\vec{\nabla}\times\left(\Phi\vec{A}\right) = \vec{\nabla}\Phi\times\vec{A} + \Phi\vec{\nabla}\times\vec{A} \tag{8}$$

$$\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} + \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right) + \vec{B}\times\left(\vec{\nabla}\times\vec{A}\right) \tag{9}$$

$$\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) = \left(\vec{\nabla}\times\vec{A}\right)\cdot\vec{B} - \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right) \tag{10}$$

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) = +\vec{A}\left(\vec{\nabla}\cdot\vec{B}\right) - \vec{B}\left(\vec{\nabla}\cdot\vec{A}\right) + \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} - \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} \tag{11}$$

With the position vector $\vec{x}$ with $r = |\vec{x}|$, $\hat{r} = \vec{r}/r$,

$$\vec{\nabla}\cdot\vec{x} = 3 \qquad \vec{\nabla}\times\vec{x} = 0 \tag{12}$$

$$\vec{\nabla}\cdot\left[\hat{r}f(r)\right] = \frac{2}{r}f + \frac{\partial f}{\partial r} \qquad \vec{\nabla}\times\left[\hat{r}f(r)\right] = 0 \tag{13}$$

$$\left(\vec{A}\cdot\vec{\nabla}\right)\hat{r}f(r) = \frac{f(r)}{r}\left[\vec{A} - \hat{r}\left(\vec{A}\cdot\hat{r}\right)\right] + \hat{r}\left(\vec{A}\cdot\hat{r}\right)\frac{\partial f}{\partial r} \tag{14}$$

$$\vec{\nabla}\left(\vec{x}\cdot\vec{A}\right) = \vec{A} + \vec{x}\left(\vec{\nabla}\cdot\vec{A}\right) + i\left(\vec{L}\times\vec{A}\right) \tag{15}$$

$$\vec{L} = -i\left(\vec{x}\times\vec{\nabla}\right) \tag{16}$$

These are some more identities

$$\vec{\nabla} \times \left(\vec{r} \times \vec{\nabla}\right) = \vec{r}\nabla^2 - \vec{\nabla}\left(1 + r\frac{\partial}{\partial r}\right)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}L^2$$