Reminder: the final exam will be held Tuesday Dec. 21, 2010, starting at 1:00 PM, in SEC 207.

This will be the last homework assignment.

Physics 507 Homework #12

Due: Thursday, Dec. 9, 2010

1. Consider the Kepler problem in two dimensions. That is, a particle of (reduced) mass μ moves in two dimensions under the influence of a potential

$$U(x,y) = -\frac{K}{\sqrt{x^2+y^2}}.$$

This is an integrable system, with two integrals of the motion which are in involution. In answering this problem you are expected to make use of the explicit solutions we found for the Kepler problem.

a) What are the two integrals of the motion, F_1 and F_2, in more familiar terms and in terms of explicit functions on phase space.

b) Show that F_1 and F_2 are in involution.

c) Pick an appropriate $\eta_0 \in \mathcal{M}_{\vec{f}}$, and explain how the coordinates $\vec{t}$ are related to the phase space coordinates $\eta = g^{\vec{t}}(\eta_0)$. This discussion may be somewhat qualitative, assuming we both know the explicit solutions of Chapter 3, but it should be clearly stated.

d) Find the vectors $\vec{e}_i$ which describe the unit cell, and give the relation between the angle variables ϕ_i and the usual coordinates η. One of these should be explicit, while the other may be described qualitatively.

e) Comment on whether there are relations among the frequencies and whether this is a degenerate system.

2. Consider a particle in two dimensions bound by a central force with potential $U(r)$. The Hamiltonian is conserved; so is the momentum conjugate to the ignorable coordinate θ, and these are in involution. Thus we have an integrable system.

Draw a sketch of the manifold $\mathcal{M}_{\vec{f}}$ embedded in the three dimensional space (x, y, p_r) which is the hyperplane of four dimensional phase space with a given p_θ. Indicate all the coordinates, including ϕ_i appropriate for $\mathcal{M}_{\vec{f}}$. Draw a trajectory in the case where there is a relation among the frequencies and give that relation, and draw another trajectory where there is no such relation. Indicate another submanifold $\mathcal{M}_{\vec{f}'}$ corresponding to the same p_θ but a slightly smaller E.

Try to mark all your objects clearly, please.